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A CRITICAL REVIEW OF THE SHORT CRACK PROBLEM IN FATIGUE

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Fatigue crack growth rate predictions based on linear elastic fracture mechanics can significantly underestimate the growth rates of smaller cracks. This study was undertaken to assess the significance of this "short-crack" phenomenon relative to a damage tolerance analysis of turbine engine components. An integrated program of experimentation and analysis was undertaken to identify the crack tip parameters that control fatigue crack growth rates for small cracks. The program began with a detailed critical review of the available literature. This report presents the results of the literature review.

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FOREWORD

This technical report has been prepared by Battelle's Columbus Laboratories, Columbus, Ohio, under USAF Contract F33615-81-C-5051. This report was developed under the first phase of the contract. The program has been administered by Mr. J. Larsen and Dr. T. Nicholas, AFWAL, Materials Laboratory.

Drs. M. F. Kanninen and B. N. Leis served as coprincipal investigators for this study. Dr. A. T. Hopper was the program manager.

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A CRITICAL REVIEW OF THE SHORT CRACK PROBLEM IN FATIGUE

1. INTRODUCTION AND BACKGROUND

Research to develop the technology needed to track the growth of inherent and service induced defects in engine components is part of the Air Force Wright Aeronautical Laboratories (AFWAL) effort to ensure integrity of engines. Such a tracking scheme will allow maximum utilization of expensive components made from strategic materials through Retirement for Cause (RFC). Essentially RFC means that components containing fatigue or creep fatigue induced damage are left in service until there is reason to remove them; e.g., a crack of near critical size. This is known as a damage tolerance approach.

One possible obstacle to the general implementation of a damage tolerance serviceability analysis for turbine engine components is the potential non-conservative behavior of short cracks. It has been found that fatigue crack growth rate predictions based on linear elastic fracture mechanics (LEFM) can significantly underestimate the growth rates of smaller cracks. One study [190]* has indicated that the higher actual growth rates of short cracks can lead to lifetimes an order of magnitude lower than are predicted by LEFM. Furthermore, the literature is full of evidence that non-conservative predictions will be commonplace when dealing with the growth of short cracks. Available data on some of the most advanced engine materials indicate that the non-conservative behavior of short cracks generally occurs only for crack sizes that are below the minimum reliable crack detection limit of currently available non-destructive evaluation (NDE) methods. So long as this is true the RFC approach based on LEFM remains valid. However, as NDE methods improve, the minimum detectable crack size may be reduced to the point that the non-conservative behavior of short cracks has important effects on

*References have been numbered by the order in which they were published. They are given in the Bibliography beginning on page 121.

damage tolerance calculations. In addition, there are many engine materials for which the behavior of short cracks has not been examined, and the behavior of short cracks using more realistic loading conditions and crack geometries is uncertain. In order to define the future impact of non-conservative behavior of short cracks on a damage tolerance approach to turbine maintenance, a more complete understanding of the limits of LEFM-based analyses applied to short cracks is required.

Although it is extensively used, LEFM has several limitations. These limitations stem from assumptions that include, among others, small scale yielding (confined flow) at crack tips. Generally, this assumption is reasonably satisfied in airframe and cold path components falling under the Airframe Structural Integrity Program (ASIP). In variable amplitude cycling an additional requirement, that current and prior crack tip plastic zone histories be similar, must also be imposed. But, this is provided for by using empirically calibrated retardation models. Consequently, LEFM has provided a viable basis in air frame and cold path applications.

Not surprisingly then, LEFM has also been explored, using laboratory specimens, as the basis for RFC in hot path applications [135]. And, so long as geometrically similar specimens have been used, LEFM has proved to be quite useful in elevated temperature studies. This is to be expected. Because the plastic zones are similar for a fixed geometry, LEFM errs to the same extent from test to test and therefore correlates data. RFC, however, cannot afford techniques that only work for a given specimen geometry. Rather, predictive techniques are needed in hot path and other situations for which LEFM errors are not self-compensating. That is, the fracture mechanics analysis must be able to deal with history dependent plastic action at crack tips in cases where the crack tip plastic zone size is on the order of the crack size.

Unfortunately, violating the contained flow requirement of LEFM is not the only complication. Engine hot path metal temperatures are in the range of 0.5 T_m (homologous temperature) and the loadings are sustained. Thermally activated time-dependent deformation occurs as do the associated damage mechanisms. Furthermore, this inelastic action occurs in component geometries which contain stress raisers (notches). Thus, the crack tip plastic zone may be contained by the plastic zone of the stress raiser. The crack-tip control condition, therefore, may be that of the notch inelastic field or the surrounding elastic domain, depending on the crack length. This geometry problem is further complicated by the fact that disk bolt hole cracks are not plane fronted; both mid thickness and corner cracking occur.

The literature relevant to the fracture mechanics of short cracks is quite extensive. A recent bibliography by Potter [175] in March of 1981, listed over 60 publications which in some way pertained to small crack growth behavior. At that same time, Battelle researchers assembled over 150 papers and reports dealing with various aspects of the problem. Since that time one major conference and several symposia have resulted in 20 additional papers. It is not surprising then that a machine-based search of Metals Abstracts turned up over 150 papers on related subjects. A similar search of the Engineering Index generated more than 175 papers.* An examination of these papers indicated that the basic causes of the small crack effect could be conveniently grouped according to the nature of the study being reported. Three categories were identified--phenomenological studies, correlative analyses, and theoretical models.

Phenomenological studies report data but contain limited analysis. These show that linear elastic fracture mechanics fails to consolidate crack growth rate behavior during the early portion of growth when cracks are small. Within this category, some papers were found that dealt with cracks in samples that did not contain notches or strain gradients. The remainder focussed on cracks growing in a gradient field, most typically due to a notch.

In the correlative analysis category, data correlation was attempted through either empirical or semi-empirical means. Generally speaking, each investigator attempted to modify LEFM to take into account that which he considered to be the source of anomalous growth rate behavior for the particular data set examined. As with the phenomenological studies, this category can be broken into groups which address either smooth specimens or notched specimens.

Papers that are contained in the third of the three categories that were identified in this study are those focussed on the development of a mathematical analysis for short cracks. Unlike the two previous categorizations,

*Papers identified were in response to key words related to short cracks. This does not necessarily mean such a paper will be relevant to this review.

no consideration is given to smooth specimens--explicit attention necessarily being given to a pre-existing crack--nor was new crack growth data collected. The papers that were reviewed concentrated on a continuum mechanics approach based on elastic-plastic constitutive behavior with little or no consideration being given to micromechanical effects. These efforts can be divided into two general subgroups: (1) simplistic efforts that are essentially plasticity-adjusted versions of LEFM, and (2) more rigorous efforts based upon the explicit consideration of residual crack tip plasticity and crack closure.

Each paper category will be examined in the ensuing sections. However, because phenomenological studies often also involved correlative attempts, these two categories are considered within a single section. While the bibliography contains over 175 references, only those papers which play a pivotal role in this review have been cited in each section. All of the papers appearing in the bibliography, however, were studied.

2. CAUSES OF THE SHORT CRACK EFFECT

This report addresses the question of why small cracks apparently grow at rates that cannot be predicted on the basis of the LEFM-based analysis methods that are successful for large (long) cracks. This statement of scope contains an implicit definition of a long crack. That is, a crack is considered to be long when its growth rate can be predicted via LEFM with the usual compact tension (CT) or center cracked panel (CCP) specimen data reported in Handbooks; e.g., reference [31]. Crack sizes that are less than this lower bound and, hence, where growth rate behavior cannot be correlated with the long crack trend obtained via LEFM, will be called short. It is important to recognize that this definition means that the short crack regime will depend upon the material, geometry, crack shape, and possibly other factors. Therefore, it is not the same for all conditions. For the present, the problem is to determine factors which cause short crack data to exhibit accelerated growth rates and to not correlate with long crack trends.

2.1 Theoretical and Physical Limitations in Fatigue Analysis

2.1.1 Mechanical Similitude

Provided the crack tip plastic zone is small compared to all other length dimensions and with respect to the distance over which the first term of the elastic stress field solution is dominant, both the size of the plastic zone and the surrounding stress field are adequately described by the stress intensity factor, K. Under these restrictions, under plane strain conditions, two cracks with equal K have the same in-plane plastic zone and stress field regardless of geometry. There is similitude.

Unfortunately, for conditions where plane strain conditions are not met, the similitude is still conditional. Of most importance, there has to be equal constraint. Otherwise, the stress parallel to the crack front σ_z, and the plastic zone are different. Thus, under circumstances of equal constraint and equal K, the response of two cracks should be the same. Therefore, two cracks should show equal growth habits. Since both K_{max} and $\Delta K = K_{max} - K_{min}$ are of relevance, it follows immediately that

$$da/dN = f_1(K_{max}, \Delta K) = f_2(\Delta K, R) \tag{2.1}$$

where $R = K_{min}/K_{max}$. Equation (2.1) is physically sound within the limitations invoked above. However, the functional forms of f_1 and f_2 cannot be derived from first principles. They must be obtained by interrogating the material through tests.

It should be pointed out that one similitude requirement has still been overlooked. This condition is that the plastic zones in the wake of the crack must also be equal in as much as they affect the local stress field through closure forces. More rigorously, similitude requires that the closure stress fields be the same. Experience from tests has shown that the latter condition is reasonably fulfilled for long cracks growing under constant amplitude, since growth rates for long cracks tend to correlate on the basis of Equation (2.1) regardless of geometry. This implies that for small cracks, Equation (2.1) is a function of the K history as well as of the current K_{max} and ΔK.

2.1.2 Metallurgical Similitude

In the derivation of Equation (2.1) it was tacitly assumed that metallurgical similitude existed. Although this may seem trivial, violation of metallurgical similitude can easily be overlooked when correlating the behavior of small and long cracks. Therefore, it is worthwhile to consider the conditions leading to metallurgical similitude.

Clearly, the material should be the same with regard to phase, orientation, dislocation density, particle density, etc. for it to respond in a unique manner to mechanically similar conditions. In the case of cracks with long fronts this condition is satisfied on the average even in multiphase materials with high crystallographic anisotropy. However, if the crack front is short (e.g., on the order of several grains), this condition will generally be violated. Apparently, the condition translates into the geometrical requirement that the crack front be long with regard to metallurgical features.

Closely related is the similitude in crack growth mechanism, which is often quoted as an important condition. However, similitude in crack growth mechanism is not a condition. It is, rather, a consequence of the material's response to the mechanical and metallurgical similitude.

Differences in crack growth mechanisms that may be considered a consequence of a breakdown in similitude can affect the growth rate, da/dN. The growth rate (crack advance over some cycle interval) is really a composite rate that reflects all operative modes and mechanisms of crack advance. Most often long crack growth in ductile engineering materials occurs by a sliding off mechanism at the crack tip. For long cracks then, crack advance is associated with Mode I loading and the dominant Mode I mechanism is reversed slip which gives rise to striations. In contrast, short crack advance may be associated with both Mode I loading and localized Mode II loading. Moreover, the Mode I mechanisms may be related to reversed slip, ductile fracture, and brittle fracture and the local Mode II mechanisms could be related to local crystallographic fracture and local shear. Thus, the character of the growth rate process can be a factor in causing the short crack behavior.

Because the environment (temperature and chemistry) affects the mechanism and rate, the environments should be the same for complete similitude. In summary, the other similitude conditions are as follows:

- small plastic zone with respect to all length dimensions (including crack front length)
- small plastic zone with respect to the distance over which the first term of the stress field solution is dominant
- equal K_{max} and ΔK
- equal closure fields
- long crack front with respect to metallurgical features.

When all of these conditions are satisfied the response of the crack will be the same. But, when any one condition is not satisfied, the bounds of validity of Equation (2.1) have been exceeded and dependencies of the type implied by Equation (2.1) become questionable.

If J is used instead of K as the mechanical similitude parameter the first requirement will be somewhat relaxed. But, all of the others remain. As discussed in more detail later in this report, J is valid only for deformation plasticity which is generally invalid when unloading from a plastic state occurs. Thus, the extension of the validity of Equation (2.1), with K replaced by J, can bring only little solace to the small crack problem. The comments that follow hold almost regardless of whether J or K is used as a similitude parameter.

2.1.3 Short Cracks at Smooth Surfaces

Provided that all similitude conditions are fulfilled, equal ΔK (and R) values give rise to equal growth rates. For a given ΔK a small crack will require a higher applied stress than a long crack. As long as the plastic zone is uniquely related to K, equal K means equal plastic zones. As a consequence the plastic zone to crack size ratio is larger for the smaller crack so that the first similitude requirement tends to be jeopardized. For very small cracks the same ΔK requires applied stresses at or close to yield and the plastic zone becomes undefined. Then the first requirement is violated and for this reason alone correlation on the basis of Equation (2.1) becomes impossible.

It has been attempted to mend this problem by applying a plastic zone correction, most notably in the case of the threshold ΔK_{th}. If ΔK_{th} has a fixed value for a certain material, it follows that

$$\Delta\sigma_{th} = \frac{\Delta K_{th}}{\beta\sqrt{\pi a}} \tag{2.2}$$

Equation (2.2) predicts $\Delta\sigma_{th}\to\infty$ for $a \to o$. This is clearly unrealistic because for a=0 one should find $\Delta\sigma_{th} = \sigma_e$ (the endurance limit). The difficulty can be avoided by introducing a plastic zone correction that increases the crack length in the manner of Irwin [5] for the fracture problem. This gives

$$\Delta\sigma_{th} = \frac{\Delta K_{th}}{\beta\sqrt{\pi(a+r_p)}} \tag{2.3}$$

For fixed ΔK_{th}, the Irwin plastic zone is constant. In that case, $\Delta\sigma_{th}$ is finite for a=0. However, for a=0 the plastic zone is certainly not r_p and therefore Equation (2.3) becomes untenable for small a. As a matter of fact, Equation (2.3) has no merits above El Haddad's equation [111] in which r_p is replaced by an arbitrary constant ℓ_0.

If a Dugdale solution is used for r_p instead, r_p will depend upon crack size, but since r_p=0 for a=0 (trivial) the solution still has little merit. From a practical point of view, Equation (2.3) would be fully acceptable if it would consolidate the data. It may do so for some cases, but, since both β and r_p depend upon geometry, it can hardly be expected that the (erroneous) equation will have generality. In addition, violation of several other similitude requirements dampens the expectations.

For short cracks the effect of local crack front irregularities and consequent local K and constraint variations are not averaged out. The shorter crack front length will further bring out the effects of microstructural variations, which may even lead to different growth mechanisms. Finally, the closure stress field will be different for the short crack because it experiences a different load history.

2.1.4 Short Cracks at Notches

For short cracks at notches (stress risers) the problem is further compounded by the notch field. At even moderate applied stress levels yielding will occur in the vicinity of the notch so that a residual stress field is introduced. The tip of a long crack would be outside this field, but the tip of a short crack is engulfed by the field. Thus, the local fields at the crack tip are different. Another consequence can be that the short-crack tip is in a displacement controlled field and the long-crack tip in a load controlled field. If the effect of this field is ignored, correlation of crack behavior on the basis of Equation (2.1) cannot be expected.

However, breakdown of the correlation is not, in the first place, due to a breakdown of similitude. Rather it is due to the fact that the condition of equal K is erroneously assumed. In principle it would be possible to account for the local field in the calculation of K and R. Provided this could be done properly, the problem of the short crack at the notch would be equivalent to that of the small crack at a smooth surface, and all comments on the breakdown of similitude given previously would apply.

2.2 The Literature on Short Cracks

Considering the almost complete breakdown of similitude for short cracks, it is not surprising that short crack effects reported in the literature show no consistency. To some extent this may be because β had to be estimated or because the wrong ΔK and R were used by ignoring a local (notch or closure) field. However, even if these artifacts could have been mended, correlations on the basis of Equation (2.1) (whether in terms of J or K) have to break down when more and more similitude conditions are violated.

For equal ΔK the nominal stress will be higher for a short crack than for a long crack. Eventually the stresses will exceed yield so that the plastic zone approaches the size of the remaining ligament. Before that ΔK becomes less and less a measure of similitude. This means that the "short crack effect" will depend upon the yield properties of the material.

Since the metallurgical similitude also breaks down for small cracks (which means that the growth rate is no longer an average taken over many grains) the local circumstances will be more reflected in the material's response. If the material is elastically or plastically anisotropic (differences in modulus and yield stress in different crystallographic directions), the local grain orientation will determine the rate of growth. A similar effect will occur in multi-phase materials. Crack front irregularities and small second phase particles or inclusions affect the local stresses and therefore the material's response. In the case of long cracks (which have long fronts) all of these effects are integrated and averaged over many grains. But, for short cracks with short fronts only the local circumstances count. It follows that the magnitude of the small crack effect will depend upon:

- the applied stress/yield stress
- the yield properties of the material
- crack front irregularity (affecting stresses and mechanism)

- crystallographic anisotropy (stresses and mechanism)
- metallurgical phases
- precipitate particles
- environment.

Several of these items are interrelated, so that the above separation is somewhat arbitrary. But, it serves to show many of the factors involved. Indeed, the literature confirms this expectation. As such, many publications are of interest, but they hardly provide any insight beyond those gained through physical arguments.

A certain number of publications simply ignore all physical reality and carry Equation (2.1) beyond all its explicit limits, sometimes attempting to patch discrepancies through artificialities such as Equation (2.3). It is hardly surprising that they meet with only limited success. Another set of publications seeks the answer entirely in the mechanical factors. Clearly, mechanical similitude has to be invoked, but it has to be invoked rigorously. Therefore, any attempts in which local fields and closure forces are ignored have no chance to succeed, and any incidental correlations are fortuitous.

Departure from stress field parameters in favor of a geometrical parameter such as CTOD certainly has merits. However, the most expedient way to obtain CTOD solutions, through a Dugdale model, is still limited to colinear cracks and small scale yielding. The big advantage of Dugdale solutions is that closure and other local fields can be accounted for, be it with difficulty. In this respect the work by Newman [202] and Seeger [29] and Fuhring [71] is very valuable as a starting point.

Naturally, mechanical similitude alone will not completely solve the problem. To the extent that crack growth is a geometrical consequence of slip, CTOD is certainly a measure of crack extension. But it is not a unique measure, because the local crack tip profile depends upon crystallographic orientation, phase, etc. Automatic averaging at long crack fronts alleviates the problem. But, for small cracks with short fronts, da/dN cannot be uniquely correlated with CTOD. The extent to which this presents a problem cannot be judged a priori. It can only be ascertained that it will be strongly material dependent. Once a satisfactory model for mechanical similitude is developed, the significance of metallurgical similitude will become obvious. It is to be hoped that it will have only minor significance for engine materials.

2.3 Difficulties in Measurement and Interpretation of Experimental Data

The value of crack growth rate, da/dN, is established by evaluating the ratio of Δa to ΔN where Δa is a suitably small increment in crack advance. ASTM Standard E647 pertaining to growth rate calculations requires a minimum Δa which is the greater of 0.25 mm or 10 times the measurement precision between readings. This is done to reduce scatter inherent in small increments of crack advance that would otherwise cloud determination of growth rate and ΔK. Because this standard applies to cracks that are large ($a/w \geq 0.2$), both the absolute and the relative error are normally held in check.

For example, for CT specimens, fatigue data are often obtained at lengths on the order of several centimeters. Thus, Δa = 0.25 mm is 1/100 to 1/500 of the crack length, but is at least a factor of 10 times the measurement precision. Travelling microscopes typically have a least count of 12.5 μm. A factor of 10 in precision would require a minimum of 0.125 mm between readings if admitted--but E647 requires a factor of 200 times the microscope's precision for long cracks. Finally, the raw data are smoothed through the use of analysis procedures that weigh individual readings based on a number of adjacent readings in calculating da/dN.

If the preceding considerations are applied to tracking the growth of short cracks, one finds that very few measurements can be made before the crack is physically long. The recommended analysis procedures would "throw away" the first several of these readings, and thereafter smooth any anomalies. Clearly then some questions must be answered regarding the accuracy of closely spaced $\Delta a/\Delta N$ readings analyzed using simple slope techniques when, for larger cracks, such procedures result in unacceptable measurement-induced scatter.

There are at least three sources of potential error in the measurement of cracks: locating the tip, determining the absolute length, and determining the relative advance of the crack over some cycle interval. The first source relates to locating a tight perhaps locally closed crack tip that may lie in a region of cyclic plasticity. This presumes that a single tip exists as it does in the long crack case. However, it is not unusual to observe many small cracks (~24 μm) all of which may grow considerably (100 μm) before a dominant crack develops. After that crack develops, its growth often involves branching up to lengths almost beyond the domain heretofore associated with so-called short crack effects. An example is shown in Figure 2.1.

Even at 50X magnification, where a crack 25 μm long appears to be 1250 μm, there may be difficulty identifying such crack tips. And, errors in this resolution, while small in an absolute sense, are large in a relative sense. Uncertainties of 10 percent are not unreasonable.

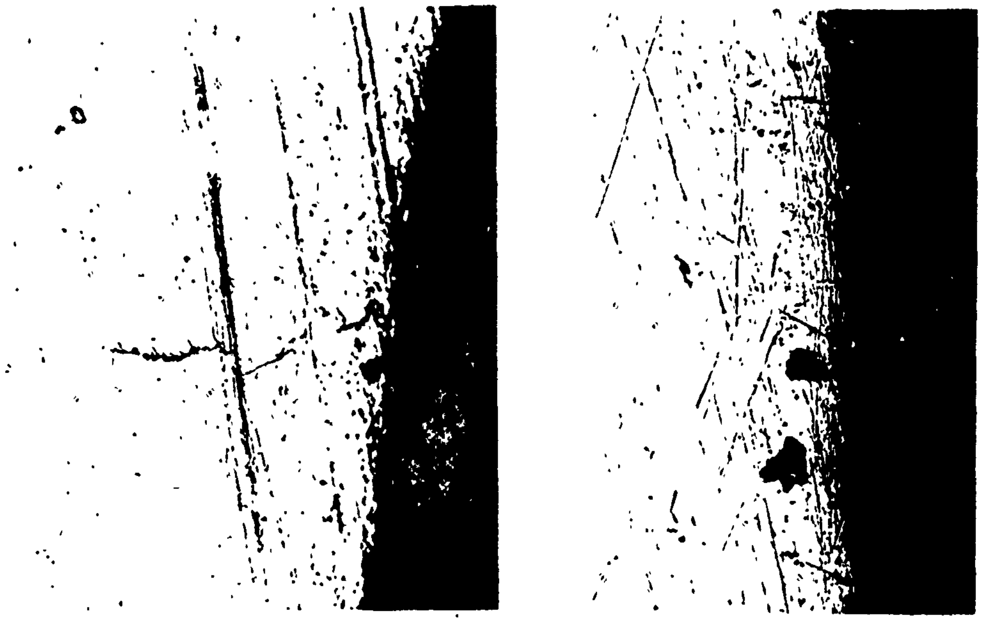
A second source of error enters through the resolution of the microscope vernier. In many studies the resolution is about 12.5 μm--about one-half of the length of the crack being measured. Clearly then a relative uncertainty of 50 percent is possible in a worst-case sense.

Finally, there is at least one further source of uncertainty which arises in measuring crack extension. Suppose that the short crack domain is on the order of 125 μm (0.005 inch). If five readings are to be made in this domain then one would seek to measure increments of 25 μm; just twice the resolution of the vernier. Again, on a worst-case basis, the uncertainty is on the order of 50 percent. Then the most probable error is $(.1^2 + .5^2)^{1/2}$ ≈50 percent for either the absolute length or the increment in crack growth.

It is easily shown that the increment in the crack advance, Δa, and the instantaneous crack length, a_i, can be related to the precision in the measurement, δ, and the error in the result, ε, as follows:

$$\frac{\Delta a}{a_i} = \frac{\delta}{\varepsilon} \quad . \tag{2.4}$$

At a level of precision of 0.01 and for an error of 0.1, Equation (2.4) indicates that a 10 percent extension of the crack is required between readings. Thus, for a 100 μm long crack, the minimum value of Δa is 10 μm.



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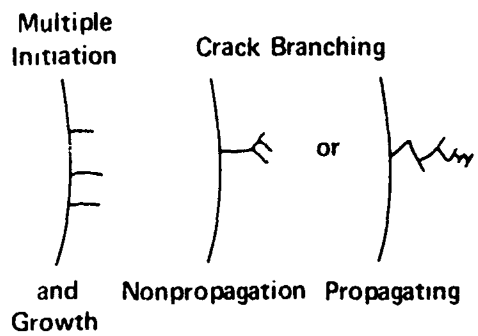
PHOTOMICROGRAPHS OF TYPICAL SHORT CRACKS



EXAMPLES OF OBSERVED COMPLEXITIES

FIGURE 2.1. MULTIPLE INITIATION AND BRANCHING (Leis and Forte [178])

However, this pleasingly small increment requires a measurement precision of .01 x 10 μm = 0.1 μm. Note that 0.1 μm is approaching the wavelength of light, and is more than a factor 10 smaller than that typically used in short crack studies.

Using more typical values one finds that $\delta \simeq 0.1$. For a_i = 100 μm and Δa = 10 μm, Equation (2.4) indicates an error on the order of 100 percent. In this respect short crack data that lie within a factor of 2 of the long crack data should not be considered as indicative of the effect.

3. PHENOMENOLOGICAL AND CORRELATIVE STUDIES

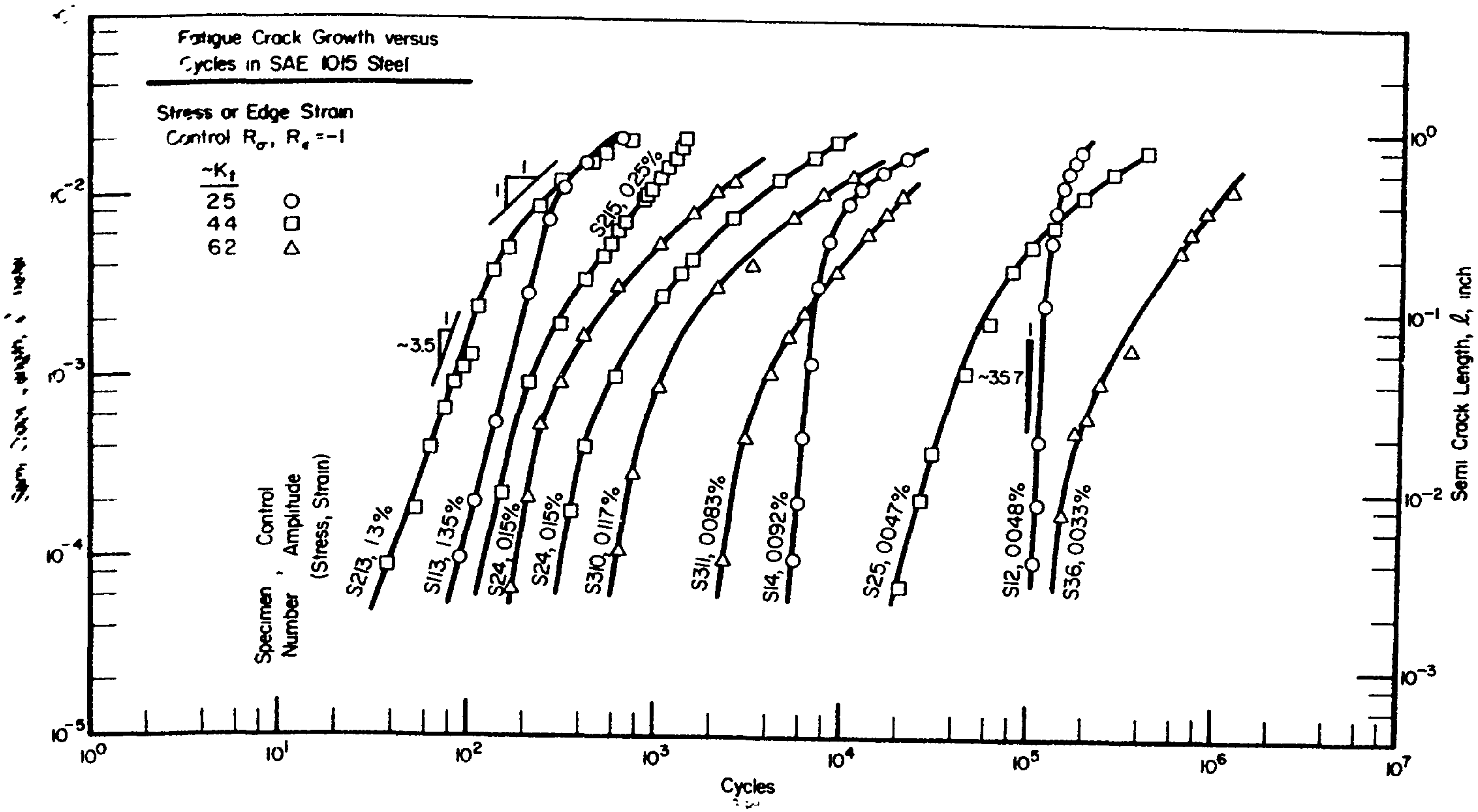
Phenomenological studies have developed data on crack length (depth), number of cycles, surface morphology, and microstructural features that develop under some controlled condition. Seldom, however, do the studies report enough data to permit a critical assessment of the results. Often, papers emphasize macroscopic observations such as LEFM correlations [94,110] or microstructural features [108,131] such as grain size. Few report both, or at least present the results independently [178,189].

Observations on growing cracks are presented in a variety of ways: crack length versus numbers of cycles, growth rate versus crack length, and growth rate versus stress intensity. Examples of each of these for the same data set are presented in Figure 3.1(a), (b), and (c). Results for near threshold conditions are most often presented on coordinates of stress and crack length for various test combinations, as shown in Figure 3.2(a). The purpose of this section is to discuss such phenomenological results with a view toward establishing to what extent each of the generic causes of a short crack effect accounts for the observations.

3.1 Near Threshold Growth of Short Cracks in Unnotched Samples

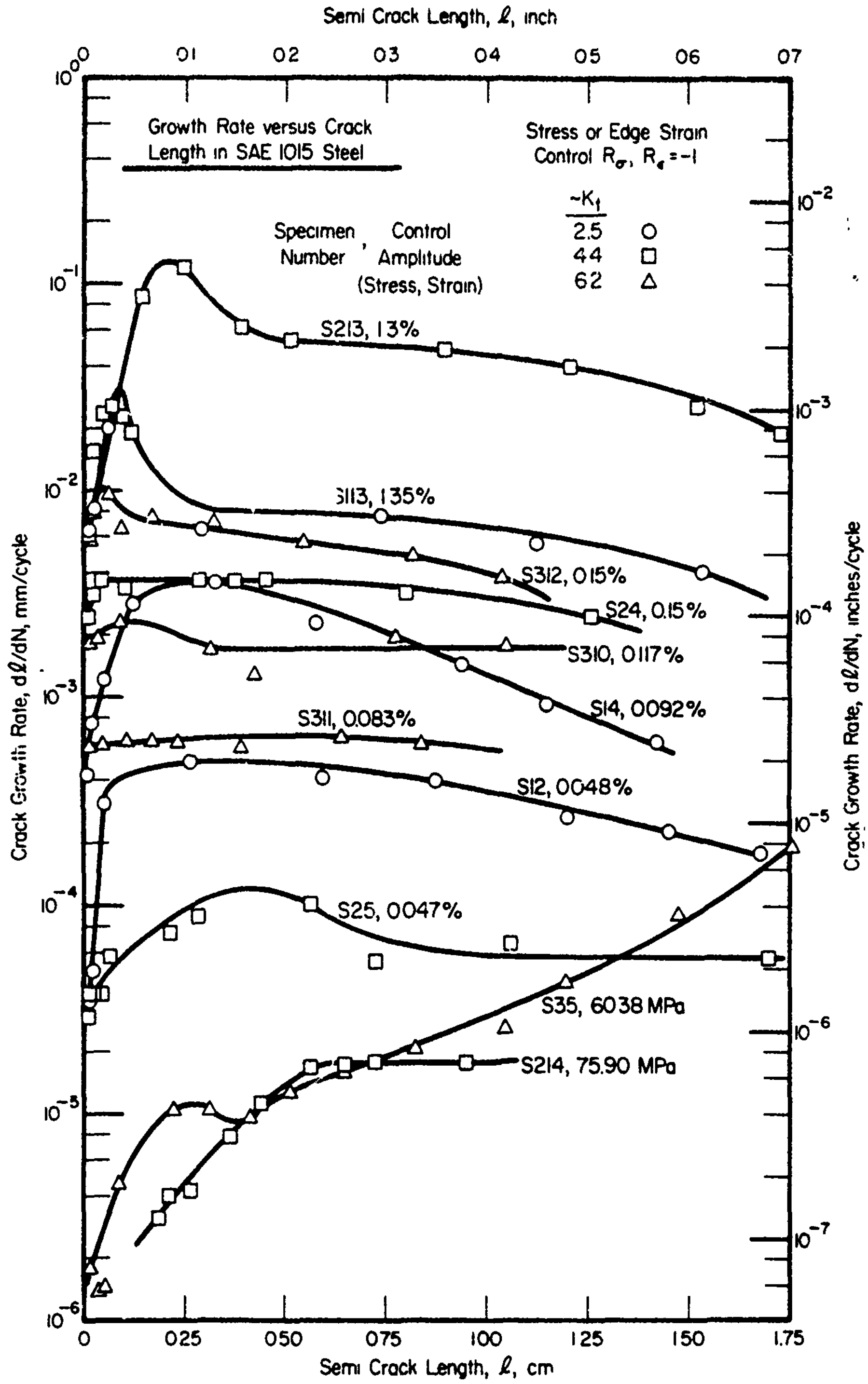
A large number of papers present information characterizing the near threshold behavior of short cracks and compare it to the threshold behavior of long cracks. Interest in this research area began and flourished in the middle to late 1950's with the work of Phillips, Fenner, Frost, and others [4,7,14]. They studied the initiation and growth behavior of cracks in very sharply notched samples. Because fracture mechanics technology had not then evolved to a level where it could be simply implemented, much of this early work involved a comparison of endurance limit behavior. Nevertheless, trends evident in the results of these early investigators are reflected in the later papers evolving from the recent flurry of interest in this same problem recast in the framework of fracture mechanics.

In the early work, differences were noted in: (1) growth periods versus dormant periods; (2) branching versus single well-defined crack tips;



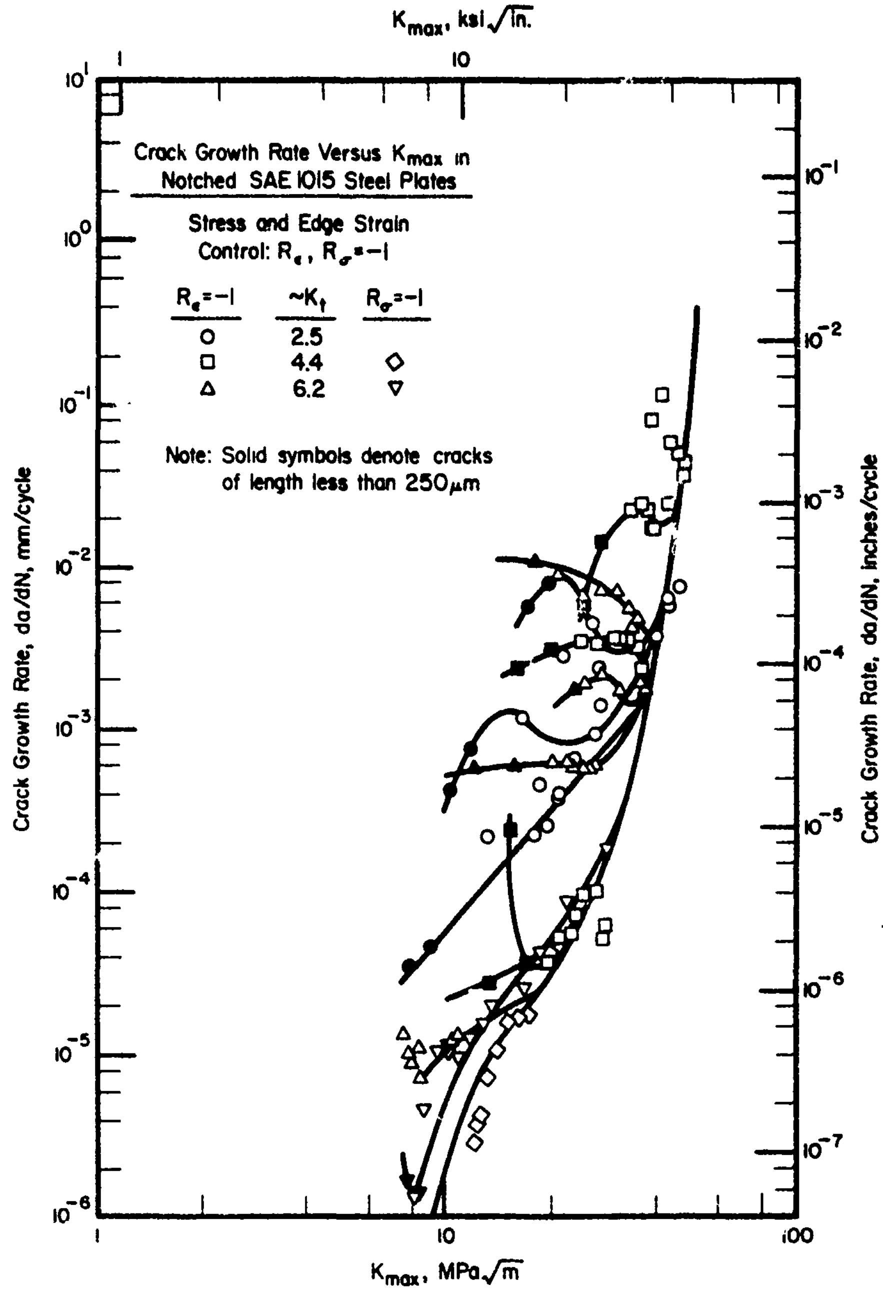
a. Crack length versus cycles

FIGURE 3.1. EXAMPLES OF CRACK GROWTH DATA
(Leis and Forte [178])



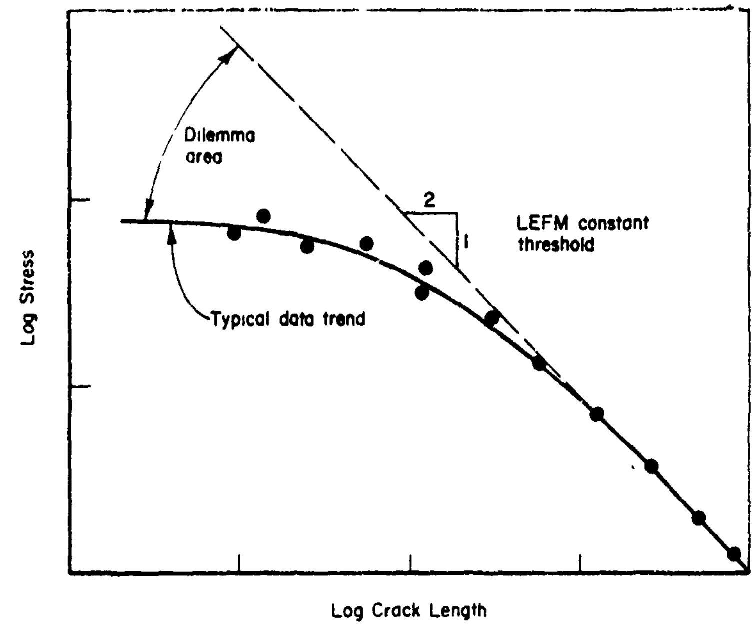
b. Crack growth rate versus crack length

FIGURE 3.1 EXAMPLES OF CRACK GROWTH DATA
(Leis and Forte [178])

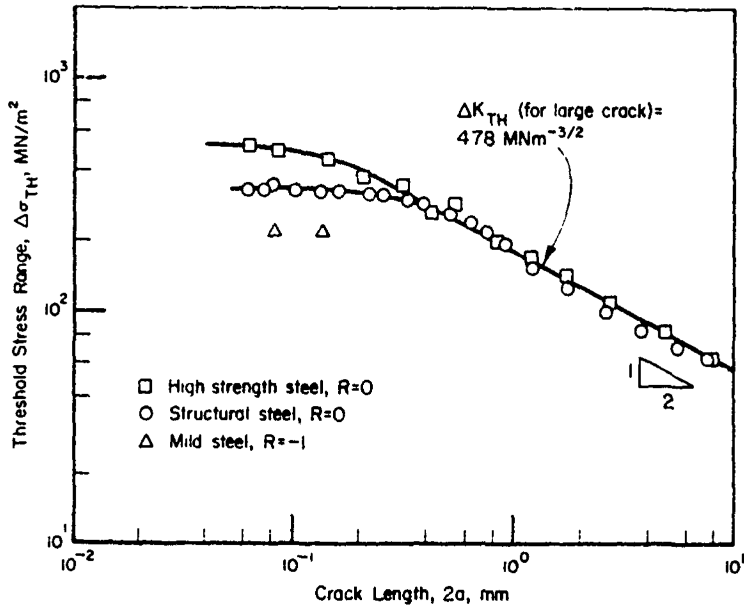


c. Crack growth rate versus stress intensity factor

FIGURE 3.1. EXAMPLES OF CRACK GROWTH DATA (Leis and Forte [178])

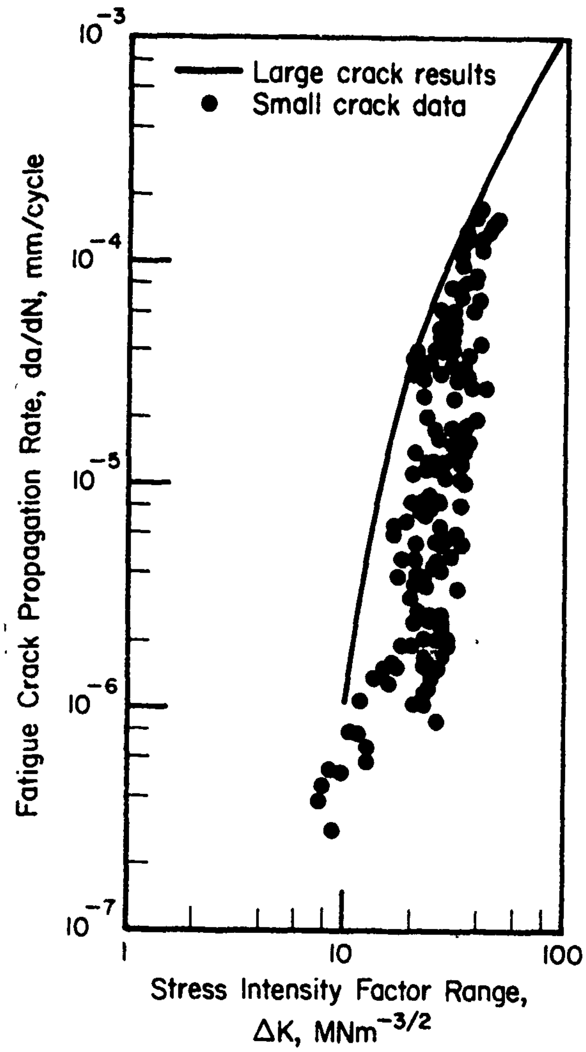


a. Schematic behavior on coordinates of log stress - log semi-crack length



b. Observed behavior on coordinates of log stress - log surface crack length (Kitagawa, et al [105])

FIGURE 3.2. FATIGUE CRACK GROWTH RATE BEHAVIOR OF SHORT CRACKS NEAR THRESHOLD



c. Crack growth rate versus stress intensity factor (Yokobori, et al [21])

FIGURE 3.2. FATIGUE CRACK GROWTH RATE BEHAVIOR OF SHORT CRACKS NEAR THRESHOLD

(3) nonplaner cracks; and (4) strong microstructural effects. Unfortunately, little attention has been paid to these differences, perhaps because this work dealt with sharply notched specimens. Not even the high initial growth rates found for physically short cracks in these studies [4] has received much attention. This is somewhat surprising in that the first paper to note a discrepancy in the thresholds for long and short cracks, that of Frost, et al [20], was based on these data. With the exception of that first paper, recent investigations have tended to view the apparently anomalous growth of physically short cracks as a novel phenomenon.

Among the first papers on unnotched specimens to indicate that the threshold stress intensity was a function of crack length is that due to Ohuchida, et al [41]*. One year later Kitagawa and Takahashi [45] showed a similar trend, although their data is rather sparse. Typical of these results are the data shown in Figure 3.2(b). Since these early papers, a number of others have shown similar trends [99,102,109,110,179]. Most notable among these are the papers by Tanaka, et al [179] and Usami and Shida [109].

The data typically exhibit the characteristic pattern shown in Figure 3.2(a). Such figures are constructed by plotting the stress associated with a given crack length at threshold conditions. It follows from Equation (2.2), repeated here for convenience,

$$\Delta K_{th} = \Delta\sigma_{th}\ \beta\sqrt{\pi a}$$

that LEFM predictions will plot as a straight line of slope--1/2 on logarithmic coordinates.** Note in Figure 3.2 that the data fall below the

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*This is the only paper we located which cited the 1971 work of Frost, Pook, and Denton [20] who first hinted at the short crack effect in a fracture mechanics sense.

**This is true, of course, only if β is independent of a/W, (i.e., a constant) and ΔK_{th} is independent of crack length. The fact that Frost et al found endurance behavior correlated with $\sigma a^{1/3}$ indicates that ΔK_{th} is not independent of a. Furthermore, only in an infinitely wide center cracked panel is β a constant. Thus, the breakdown of the trend in the slope of -1/2 is to be expected.

LEFM trend for short cracks. These data tend to asymptotically approach a stress whose value is equal to the endurance limit. In this respect, one would expect metallurgical and other material properties to correlate for long and short cracks. Finally, note that this endurance limit is consistently overestimated by LEFM--as $a \to o$, $\Delta\sigma_{th} \to \infty$ for constant ΔK_{th} so that LEFM conditions are violated. In this respect, LEFM should not be used near the endurance limit.

Two studies have shown that in high strength materials, the LEFM trend of Figure 3.2 is followed for all cracks greater in length than the size of an inclusion ($\sim 10^{-2}$ mm) [128,148]. Moreover, there is one set of data which has the opposite trend to that of Figure 3.2. Data developed by Yokobori et al [21], presented in Figure 3.2(c), shows a short crack threshold stress greater than that for long cracks. In discussing these data, Yokobori [120] emphasized that the initiation and microcrack growth behavior were much different than for long cracks. This involved a ragged, poorly defined crack front. In view of data developed by Schijve [149], multiple initiation, branching, and a poorly defined crack tip will produce depressed growth rates as compared to a well defined crack of equivalent length. Apparently, a poorly defined crack front, and perhaps closure effects on R, explain this trend.

Because of its importance a few words about closure effects on long and short cracks are appropriate. For long cracks, growth has been observed to be a strong function of stress ratio, R [31]. The reason for this is as follows. Because the plastic zone ahead of the crack is a function of crack length, the residual plasticity in the wake of the crack, and consequently the opening load, will also depend on the crack length. Thus, the effective range of K_{th}, ΔK_{th}^{eff}, defined for the increment of load during which the crack is open, may be substantially less than the total range; i.e., ΔK_{th} [58]. When ΔK_{th}^{eff} is used in place of ΔK_{th}, the LEFM ΔK_{th} trend of Figure 3.2(a) will shift to lower values of stress and crack length.

For a short crack contained within a constrained plane stress surface yield zone, localized closure could similarly reduce the effectiveness of ΔK_{th}. The same statement can be made for small active regions of growth which develop at low growth rates along broad inactive crack fronts in long cracks. In both of these cases, crack growth may occur under locally reduced levels of ΔK_{th} due to localized closure. However, because the effect of localized closure is averaged, in the second instance over a long crack tip, it is unlikely that the ΔK_{th}^{eff} which actually controls local growth could be sensed in bulk measurements.

There is additional evidence that ΔK_{th}^{eff} is related to the data trends of Figure 3.2(a). If a constant value of ΔK_{th}^{eff} is used in place of ΔK_{th}, the trend predicted by ΔK and closure (still LEFM) would be concave down. Both the linear and nonlinear (ΔK_{th}^{eff}) trends arise from LEFM considerations. Finally, one study has shown that a minimum size of active reversed crack tip plasticity is required to sustain near threshold growth [55]. The implication here is that ΔK_{th} and closure influence the size of the active plastic zone. For this reason ΔK_{th}^{eff} but not ΔK_{th} would be expected to correlate crack growth over a range of crack sizes. But, it must be emphasized again that in the limit as $a \to o$ the use of LEFM is inappropriate.

Many studies [14,23,30,89,101,104,124,146,170] indicate the threshold for long cracks depends strongly on metallurgical features. Experimental results [86] show that values of the threshold decrease with increasing yield strength in some ferritic-pearlitic steels. Other studies show a varied dependence on grain size in steels and nonferrous materials [54,121]. Somewhat in conflict with this are the observations for these materials which show fatigue resistance increases with increasing cyclic strength and decreasing grain size [86].* In contrast, certain nonferrous materials exhibit the reverse (but anticipated) trend. Results for these materials indicate that small reductions in the threshold develop as cyclic strength is decreased. Why these radically different trends develop remains uncertain. Attempts have been made to explain the results for the nonferrous materials in terms of an LEFM-based measure of crack opening displacement (COD) [65]. However, such explanations are inconsistent with the results for steels. Unfortunately, the expected trends observed for the nonferrous

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*Recall that the endurance limit stress is the asymptotic bound as crack length goes to zero in Figure 3.2. One would thus expect this trend curve to move consistently, i.e., one would expect resistance to cracking would either improve or degrade independent of crack length. Instead, one finds that as the LEFM trend moves up, the limit for small cracks moves down.

materials probably are erroneous because the confined plasticity limitations of LEFM are violated for these lower strength materials. Valid application of LEFM would yield the same trend as seen for the steel. For this reason, comparisons of trends in ΔK_{th} for ferrous and nonferrous materials are clouded by the uncertain validity of stress intensity calculation under these conditions.

Further insight into the divergent effects of microstructure on fatigue limits and growth thresholds can be gained by considering microstructually induced differences in initiation behavior at endurance conditions. There is experimental evidence that the relationship between crack size and stress is linear under endurance conditions [77]. LEFM predicts a hyperbolic functional form, suggesting that LEFM has no fundamental role in correlating these variables, at least for the case examined. A second study showed that the character of the initiation process at endurance depends strongly on grain size (hardness) [180]. Lower strength low carbon steel was observed to develop a single dominant crack, whereas, the more fine grained higher strength materials tend to develop branched cracks. Branching reduces the value of ΔK and the size of the plastic zone at the crack tip. Consequently, this material would be expected to have an increased resistance to microcrack propagation. This would indicate that endurance will increase with grain size for small cracks.

With respect to threshold behavior, literature reviews show that for long cracks, threshold decreases with increasing grain size [23,101]. This, of course, is opposite to the response to endurance. However, there are few physical reasons to expect threshold and endurance to respond in the same way to grain size changes.

The fatigue limit has been defined as the stress below which microcracks do not propagate to a critical size [18]. Applying this definition to the schematic of Figure 3.2(a), the asymptote of the trend for decreasing (but nonvanishing) crack length would be the fatigue limit. The data shown in Figure 3.3, for cracks in the near threshold regime, indeed tend toward this limit. This tendency is anticipated if the threshold and the endurance limit are related concepts. However, it is difficult to view these concepts as being related in a fundamental way given the fact that

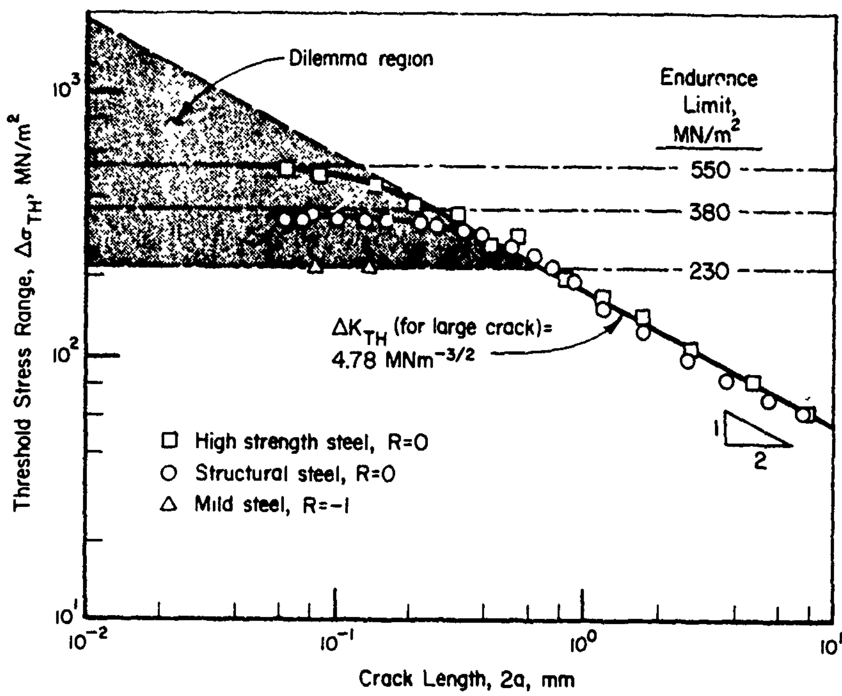


FIGURE 3.3. FATIGUE CRACK GROWTH RATE BEHAVIOR OF SHORT CRACKS NEAR THRESHOLD
(Kitagawa et al [103])

metallurgical changes which tend to increase long crack thresholds through grain size or other strengthening mechanisms tend to decrease the fatigue limit. The question thus arises--is the trend in Figure 3.1 or 3.2 coincidental, or does it develop because the threshold and the endurance limit are measures of the same material property expressed in different terms? The question can be explored from two points of view. The first brings the definition of endurance limit into question: is the definition of the endurance limit as the stress below which microcracks do not propagate inappropriate in general? The second relates to the question: is there a transition in the mechanisms that control microcrack initiation and growth with decreasing crack size? The literature does not permit a conclusive response to either of these questions. The significance of the answers appears to be of consequence, depending on the material and stress level, as follows.

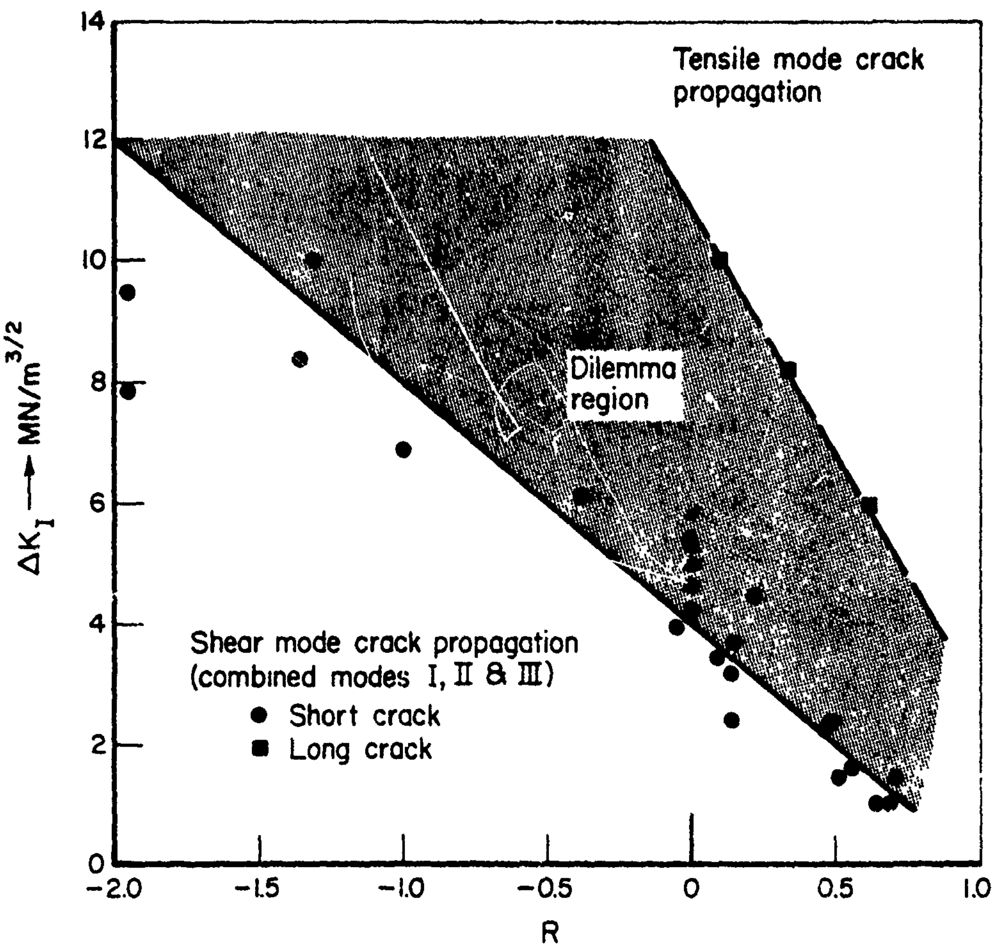
It is certainly true that for many materials long cracks grow by a predominantly striation forming mechanism near the threshold [23,101]. If for these materials, shorter cracks involve other Mode I mechanisms or local Mode II mechanisms, the growth of short cracks would occur when the long crack growth rate via striation formation is negligible. Thus, LEFM would overestimate the value of ΔK_{th}, the effect observed in Figures 3.2 and 3.3. Unfortunately, data which show this effect seldom include a discussion of fractography. Thus, one can only postulate, without conclusive experimental support, that this difference between long and short cracks is due to a transition in growth mechanisms.

There are also data which show that the near threshold growth of long cracks involves mechanisms besides striation formation for some steels [18]. Thus, for certain materials, (e.g., high strength steels), the dilemma of Figure 3.3 cannot be explained simply in terms of multiple mechanisms. For some of this class of materials, near threshold growth of long cracks is argued to be environmentally controlled [86]. Crack length becomes an obvious factor in such cases. Wedging, which may develop through oxide formation and buildup on crack faces, depends on the access distance to the environment. To be of consequence, such buildup would need to be on the order of the crack tip opening displacement (CTOD) so that near threshold conditions would be the first to be effected. Thus, a transition in mechanisms due to environmental enhancement can be expected as crack length decreases. Again higher growth rates would be predicted for the same crack length as compared to LEFM, in the absence of an accurate environmentally assisted cracking (EAC) model. This is not the fault of LEFM. Rather, it is a limitation in LEFM as currently implemented.

The literature also indicates that even when multiple crack growth mechanisms are not of consequence, differences in long and short crack threshold behavior remain. Data developed by Helle [184] are presented in Figure 3.4(a) to illustrate this point. Notice from the figure that short and long crack stress intensities for true Mode I growth differ substantially. Significant differences still remain when this comparison is made on the basis of K_{max}, as is evident in Figure 3.4(b). This suggests that macroscopic closure is not solely responsible for the difference in values of ΔK or K_{max} for long and short crack transitions to Mode I cracking. Helle suggests this difference exists because of anisotropic flow. He also infers the significance of closure developing only after some crack growth. Furthermore, he suggests that the mechanics of short crack extension differs from that for long cracks beyond closure because they retain a consistently straight crack front for his copper based alloy.

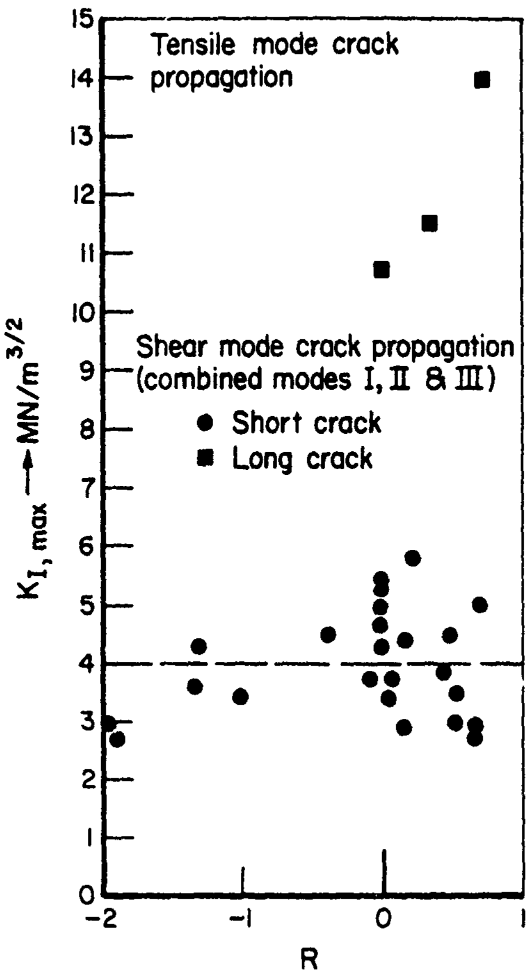
Helle's observation on closure is consistent with observations discussed earlier in this report. However, his second observation appears to be inconsistent with other studies. For example, some results suggest that the crack front of physically small cracks involve mixed modes, irregular fronts, and crystallograhpically controlled directional cracking [189]. An example is shown in Figure 3.5.

Helle's data have interesting implications regarding long crack thresholds and the fatigue limit based on a threshold stress for propagation. With reference to Figure 3.4(a) note that, as pointed out by Schijve [177], even though only Mode I is involved, ΔK by itself does not uniquely characterize the threshold for this material. Because the growth of both long and short cracks occurred by the same mechanism, the difference between these long and short cracks must be ascribed to a mechanics effect. That is, for these data the mechanics of long and short crack growth differs when correlated



a. ΔK_I values of small fatigue cracks at the transition from shear mode cracking to tensile mode cracking

FIGURE 3.4. DEPENDENCE OF SHEAR TO TENSILE MODE CRACK GROWTH ON STRESS RATIO (Helle [184])



b. $K_{I,max}$ values for the crack at transition corresponding to part (a)

FIGURE 3.4. DEPENDENCE OF SHEAR TO TENSILE MODE CRACK GROWTH ON STRESS RATIO (After Helle [184, 186])

500X, SEM

FIGURE 3.5. FRACTOGRAPH FEATURES ASSOCIATED WITH CRACK INITIATION AND MICROCRACK GROWTH (Leis and Galliher [189])

Note rough fracture surface and absence of any evidence of a defined crack front. Long cracks show a defined crack tip and a much smoother surface (2024 T351 aluminum notched plate with K_t = 2.56, cycled at net section stress of 255 MPa and R = 0.01) using rudimentary LEFM concepts. Therefore, it appears to be questionable to infer a threshold for these data for nonpropagation of cracks at fatigue limit conditions, based on rudimentary LEFM calculations.

The essential difference between the two situations just discussed is that Helle's material apparently transitioned to a stable opening mode quickly at near threshold conditions, whereas other materials do not. While microstructural features appear to play a critical role, the literature suggests that such features may be aligned or have characteristics shapes which can either enhance or retard slip and microcrack initiation and growth [57]. Whether a particular feature enhances or retards these processes appears to depend on environment [198]. However, microstructural effects on mechanics may also be significant. For example, initiated microcracks have been observed to branch in certain microstructures [180] leading to a reduction in stress intensity (crack arrest) and a fatigue limit condition.

Depending on the number of branches, the mechanics of a branched crack that occurs near the fatigue limit suggests that the crack driving force is much less than that for a single crack of comparable length [51]. To this end, microstructure may cause a short crack to stop growing at a stress level below that anticipated based on the single crack tip commonly associated with long crack behavior. Only if a highly irregular front that significantly reduced the value of K_{th} developed for long cracks could one expect irregular branched crack thresholds and long crack thresholds to be comparable. However, based on geometric similitude, the irregularity would have to be on the order of the crack length. (Such a test result would certainly be considered invalid in view of current standards for crack growth rate studies such as ASTM E647).

To summarize, it is unlikely that short and long cracks growing under near threshold conditions would do so under comparable values of stress intensity factor. Closure is clearly a factor, and is capable of producing the data trend shown in Figure 3.2. Differences in mode and mechanism of growth between long and short cracks are also factors. It appears that metallurgical features play a major role in this regard. Likewise, metallurgical features tend to control whether or not branching occurs, and to what extent. Finally, the trend with smaller cracks is toward a threshold stress that approaches the fatigue limit of the material. However, a physical basis or mechanical basis is lacking for the prediction of endurance limits from LEFM and long crack data. Thus, this trend does not infer any fundamental tie between the concepts of threshold and endurance limit. Any parameter that arises from an equation between these concepts is therefore empirical, and is a reflection of the factors that cause differences between short and long cracks. Unfortunately, because many of these factors are geometry dependent (e.g., closure), the empirical relationship is likewise geometry dependent.

3.2 Finite Growth Rate Behavior of Short Cracks in Unnotched Samples

Only a few investigators have examined the finite growth rate behavior of short cracks in unnotched specimens [43,60,95,128,131,154,185]. Dowling [60] found that short cracks grow somewhat faster than do long cracks in A533 B steel. His results, reproduced in Figure 3.6, are correlated on the basis of ΔJ. ΔJ is used in lieu of ΔK because his tests involved fully-reversed displacement controlled cycling, at levels which caused gross plastic flow. These data indicate a short crack growth rate effect. But is it real or merely scatter in the data? The following discussion explores sources of uncertainty in the data and analyses to assess this possibility.

Limited observation has shown that crack growth rate can be consolidated on the basis of ΔJ [46]. Limitations to the validity of this correlation exist [134], as do limitations to the validity of J. However, for the data shown in Figure 3.6, virtually all limitations are met provided that Dowling's definition of ΔJ is valid. These data were developed from measurements at 100X on surface replicas made periodically throughout the test. During the period of cracking that does not correlate with long crack data, typically only two or three replicas were available for study. Growth rate was obtained from a simple slope calculation based on these results. Because the crack length was small, and because during this period a relatively small amount of crack advance occurred, even a small absolute error in measuring crack length results in a large relative error in da/dN.

There is also uncertainty in the value of J, beyond that associated with uncertainty in crack length, which may tend to underestimate the

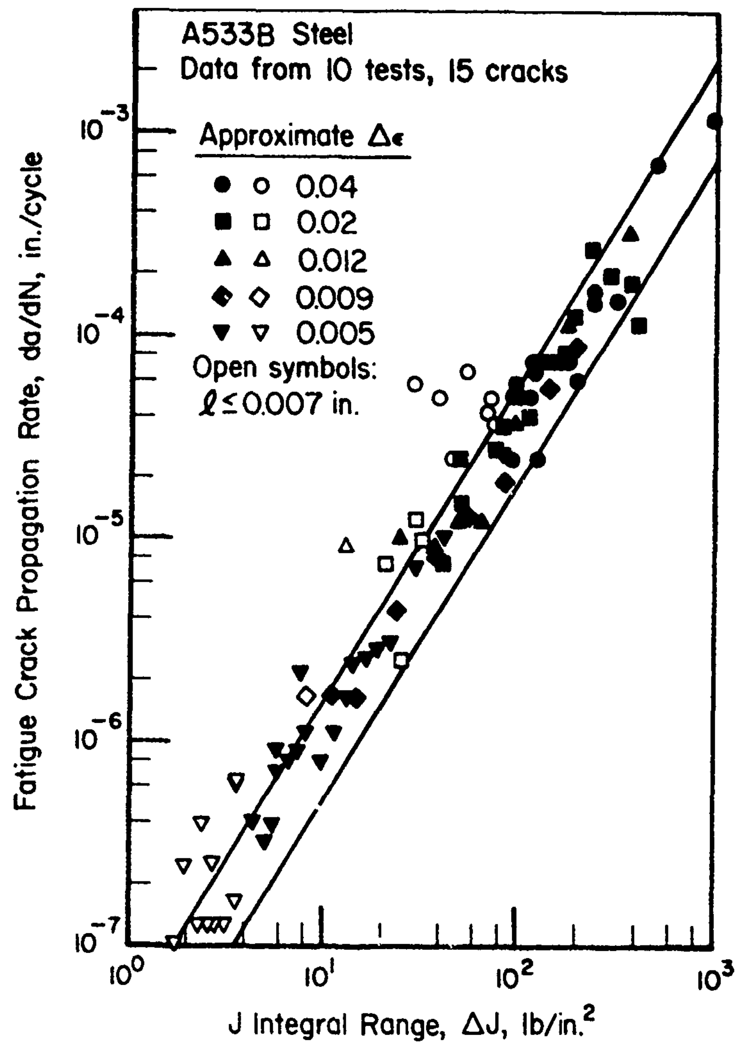


FIGURE 3.6. COMPARISON OF SMALL CRACK DATA WITH SCATTERBAND FROM TEST RESULTS FOR ORDINARY AND LARGE-SIZE SPECIMENS (Dowling [60])

magnitude of J. This error, however, is the same for any J calculated. As a result, the value of ΔJ will not be in doubt. This fortuitous circumstance does not help the fact that closure of a small crack cannot be detected readily in terms of far field load response. This, coupled with the fact that an array of cracks (not branched) such as developed in Dowling's test under displacement control, results in a decreasing K field*. This suggests that until one dominant macrocrack develops ΔJ probably is not an adequate measure of crack driving force.

In view of this discussion there are numerous sources of uncertainty and as noted by Dowling, "the method used in obtaining ΔJ . . . can be regarded only as a rough estimate" [60]. Scatter may therefore be a consideration--nevertheless, the data do indicate definite trends which suggest short cracks grow faster than their longer counterparts.

It should be noted that the short crack behavior reported by Dowling appeared to involve a well defined unbranched crack front, and an essentially planar crack. Thus, in his test material, the growth probably was due to Mode I cracking and mechanisms similar to those which control growth in long cracks. Additional microscopic factors are therefore probably not a factor in these data. Because gross plasticity is involved, it is unlikely that grain boundaries served to confine crack tip flow either. It would appear then that the limited differences in Dowling's short crack growth rate behavior are not associated with microstructural features and differences in crack growth mechanism. Thus, it appears that differences between short and long cracks are most likely due to the inability of the "rough estimate" of ΔJ to reflect the closure behavior of small cracks. Nisitani and Kage [81] report data that suggest local closure in short cracks has a very significant effect on growth rate.

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* A decreasing value of ΔK implies that the elastic component of ΔJ decreases. For Dowling's tests ΔJ is controlled by the plastic component of J. Whether or not ΔJ will change substantially due to an array of small cracks is unknown. Thus, the influence of the array on the absolute value of J and crack closure (and thus on the operational definition of ΔJ) remains in question.

A number of ongoing studies by Morris et al at Rockwell Science Center have focused on the growth behavior of small cracks growing in 2219 T851 aluminum alloy [108,131]. These studies deal with cracking initially contained within a grain. It is argued that crack tip deformation behavior and closure are controlling parameters in this stochastic growth process. They emphasize an incubation period during which the material at the crack tip must be plastically deformed before growth begins through the grain boundary into the next grain. In this respect, their view is consistent with the 1976 suggestion of Seika et al [55], who showed experimentally that a minimum amount of reversed plasticity at an opening crack tip is necessary for crack extension.

The work of Morris et al uses CTOD* as a measure of crack tip plasticity, coupled with an empirical measure of closure to accommodate closure effects. Their experiments indicate that growth rate is reduced shortly after passing a grain boundary, but resumes at a comparable rate prior to meeting the next boundary. This effect is evident in Figure 3.7. Their measurements show that the SCOD measured is essentially equal to the limiting value anticipated from linear elastic calculations. In their studies SCOD correlated trends in growth rate behavior for small cracks. A similar observation had been made some time earlier in terms of COD for long cracks [55].

Since grain boundaries are viewed by Morris et al as crack barriers, one would expect growth rate to decrease as the grain boundary is approached and increase as unblocked virgin material is entered. Instead their data show the growth rate increasing to the boundary, and decreasing for a small distance thereafter [131]. Thus, although their empirical model can be calibrated to predict observed trends, the physical basis of the model appears to be inconsistent with their own observations. A more satisfying physical model

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* Although Morris et al uses the term CTOD, they actually measured surface (near tip) crack opening displacment (SCOD). It may be more precise to use SCOD or perhaps surface crack tip opening displacement (SCTOD).

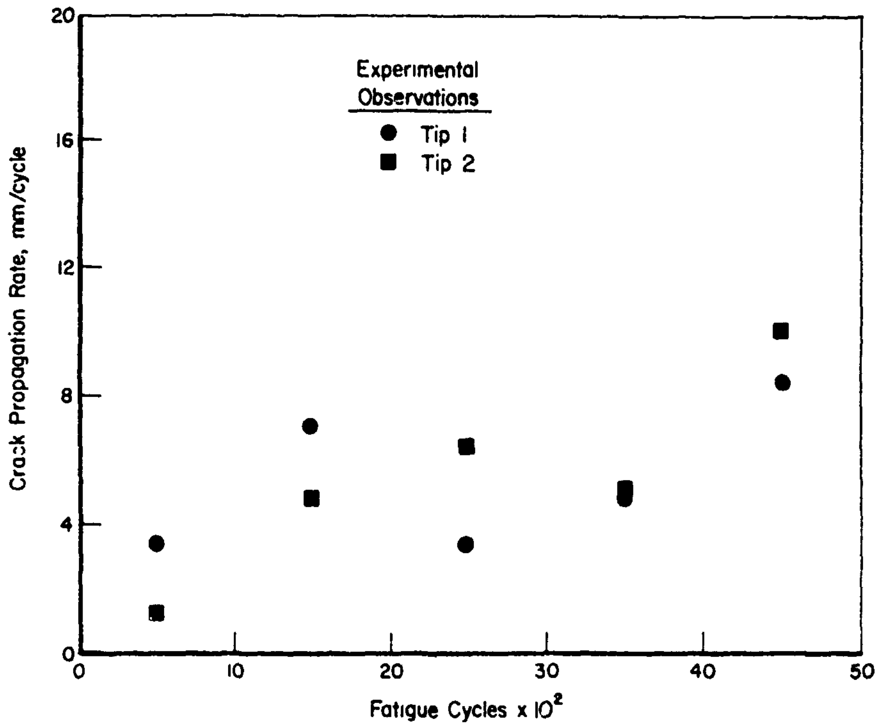

FIGURE 3.7. GROWTH RATE FOR AN INDIVIDUAL CRACK FOR BOTH SURFACE CRACK TIPS. THE DECREASE IN PROPAGATION RATE OCCURRED WHEN EACH TIP CROSSED A GRAIN BOUNDARY (Morris, et al [131])

would have the growth rate be proportional to the local stress. The highest stresses would occur at grain boundaries as dislocations build up at the boundary. In such a physical model, the decrease in rate as the boundary is passed would be due to crack growth through the matrix of the grain where the stress would be lower. Grain to grain compatibility would influence the stress concentrating at grain boundaries. Since such compatibility differs on the surface, as compared to below the surface, any similar model including that of Morris et al would be sensitive to differing compatibility conditions (e.g., crack depth).

The studies of Morris et al are significant from a research view point. However, they only report surface results for apparently shallow cracks. This suggests that in practical problems, surface treatments like cold work, may confound implementing their findings. It also suggests that if subsurface ties to the surface crack tips differ from those that they conceived, different trends and conclusions may develop. Furthermore, their use of very thin specimens generates plastic flow (SCOD) that is unique to the degree of grain to grain compatibility and plane stress developed in their specimens. These aspects do not limit the significance of their results for the very restricted situation they examined. But, they may limit the extent to which their observations can be ascribed generality.

In a study similar to that of Morris et al, Hack and Leverant [127,181] have examined SCOD in Ti-6Al-4V and HY-130 steel under a variety of conditions. Their study, which was initiated at about the same time as the work of Morris et al, uses essentially the same experimental procedures. However, it tends to deal with substantially longer cracks (50-500 μm). LEFM with the usual Irwin plasticity correction was found suitable to predict SCOD behavior.

One aspect of Hack and Leverant's studies deserves further comment. The study dealing with the steel showed that a strong closure effect arises in the presence of an artificially induced residual stress. A similar surface residual stress develops naturally when the surface, under plane stress conditions, yields in tension and is confined by the elastic plane strain core. Since this situation commonly develops at strain levels below gross yield, this finding has particular significance relative to closure effects, and consequently the propagation rate of small surface cracks. Indeed, residual surface stresses are inherent in all tests at positive values of R, and so may control microcrack closure in experiments such as those of Morris et al just discussed. The data developed by Nisitani and Kage [81] indicate that closure effects develop very near to the crack tip. For this reason, the SCOD measurements of both Morris et al and Hack and Leverant may substantially underestimate the true extent of microcrack closure. Moreover, if closure is due to a surface plane-stress-state confined by a plane-strain elastic core, results developed for closure become specimen and crack geometry dependent. This dependence is also suggested in results developed by Schijve [153] and others [42,147] who have shown that both the cracking mode and shear lip formation are factors in closure.

Lankford et al have studied microcrack initiation and growth in a hard steel and a superalloy [43,95]. Data developed for the 4340 steel indicated the growth process is discontinuous and transgranular as shown in Figure 3.8. Crack growth rate measurements were found to correlate with LEFM provided that the debonded region at inclusions associated with initiation was included in the crack length. Here, the successful use of LEFM is credited to the smaller plastic zone in high strength steels as compared to the plastic zone size in lower strength materials for the same ΔK. Thus, LEFM is valid for smaller crack sizes in hard steels as compared to low strength materials. At crack lengths less than 60 μm, growth is intermittent, similar to the nonpropagation periods in the early growth observed in the studies of Frost, et al [4]. Obviously, intermittent growth will require analyses which do not assume continuous growth in calculating growth rates.

All data developed for the IN100 superalloy reported in various papers are summarized in an AFML report [95]. Included are both ambient and elevated temperature results. At room temperature, the results showed that cracks "hang up" at grain boundaries causing intermittent transgranular growth, much the same as indicated in the data developed by Morris et al. However, unlike the results for the steel, this smaller grained material gave less marked periods of hesitation leading to a much smoother growth rate behavior. Except for that portion of the crack growth record which occurred during the period of hesitant growth, the data developed are well consolidated

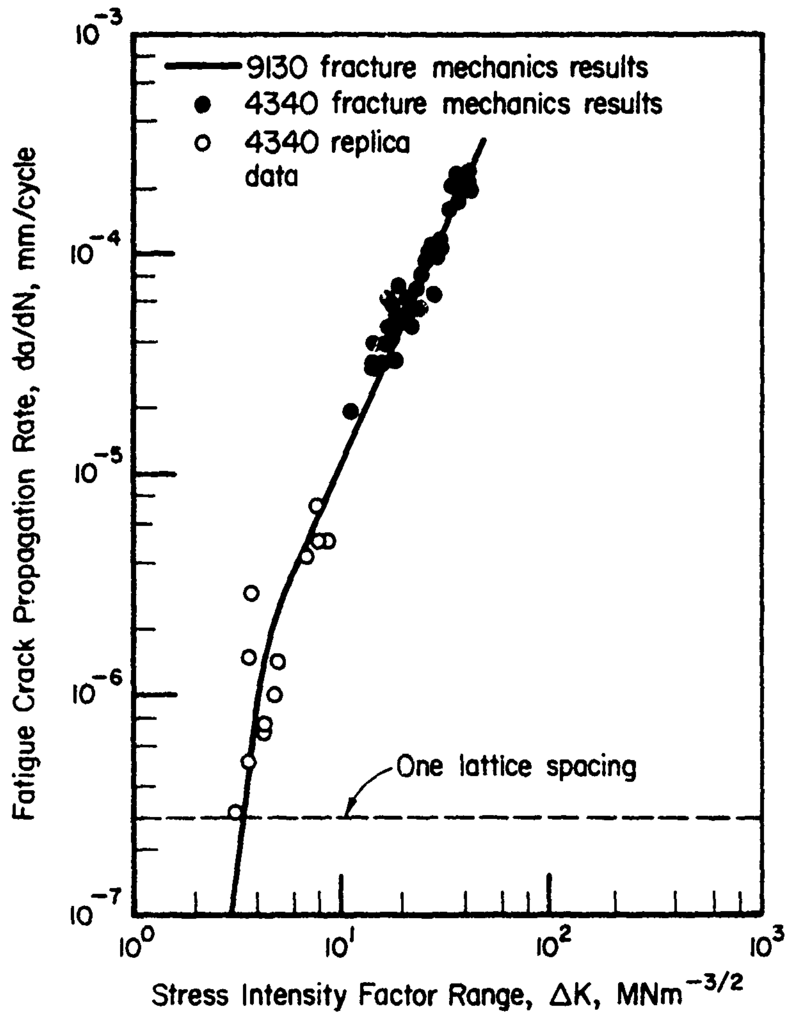

FIGURE 3.8. FATIGUE CRACK GROWTH RATE VERSUS STRESS INTENSITY FOR A HIGH STRENGTH STEEL (Lankford, et al [95, 43])

by LEFM. As shown in Figure 3.9, the hesitation does not give rise to large discrepancies. This is consistent with the observations of Gangloff [128] who observed that at room temperature, small-crack (100 μm) growth-rate data for another superalloy, Rene 95, were also well consolidated by LEFM. Results developed by Lankford et al at elevated temperature (650 C) showed intergranular cracking. Growth rate data did not exhibit anomalous growth because it was forced to form a continuous semicircular front. Obviously, because the growth is intergranular, there was no evidence of grain boundaries "hanging up" growth.

The data for the steel show that the initiation mechanisms and the inclusion size can have a significant effect on short crack behavior. But, they have only a nominal effect on the growth rate of long cracks. The data also show that the cracking mode and the shape and continuity of the front are controlling factors for physically small cracks.

More recent work has resumed Dowling's studies of large scale plasticity in low cycle fatigue (LCF) specimens. This work, reported by Starkey and Irving [185], examined ferritic and pearlitic irons. As found by Lankford et al, when the length of the defect (microporosity) is included in the crack length, adequate consolidation of the data occurs on the basis of ΔJ for cracks greater than 150 μm deep.* But, as was the case with Dowling's data, the same uncertainties in using ΔJ exist.

The final data set to be explored is a limited one recently developed by Taylor and Knott [154]. They tested bend samples made of a coarse-grained nickel-aluminum-bronze alloy and discussed data for materials having different grain sizes.** Using the data shown in Figure 3.10, they demonstrate that so-called short crack behavior may develop in physically long cracks (up to 400 μm or several grains in their material). They suggest that the transition to long crack growth follows only after the crack has grown

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* 150 μm is about the average size for the microporosity for their materials.

** Because grain size influences strength level, there is no clear-cut tie between the data discussed and grain size. This is because plastic zone size for a given crack length will vary inversely with strength. Thus, both the microstructure and the mechanics change with grain size.

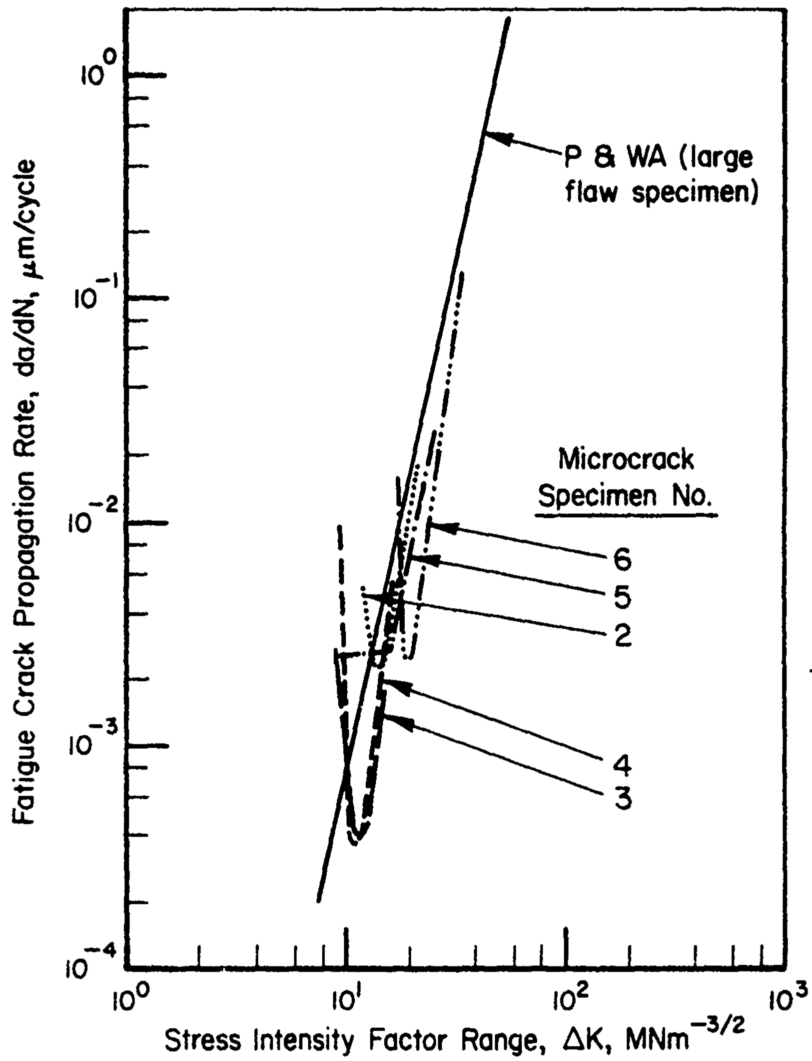

FIGURE 3.9. CRACK GROWTH RATE VERSUS STRESS INTENSITY FOR SEVERAL IN-100 SPECIMENS (Cook, et al [95])

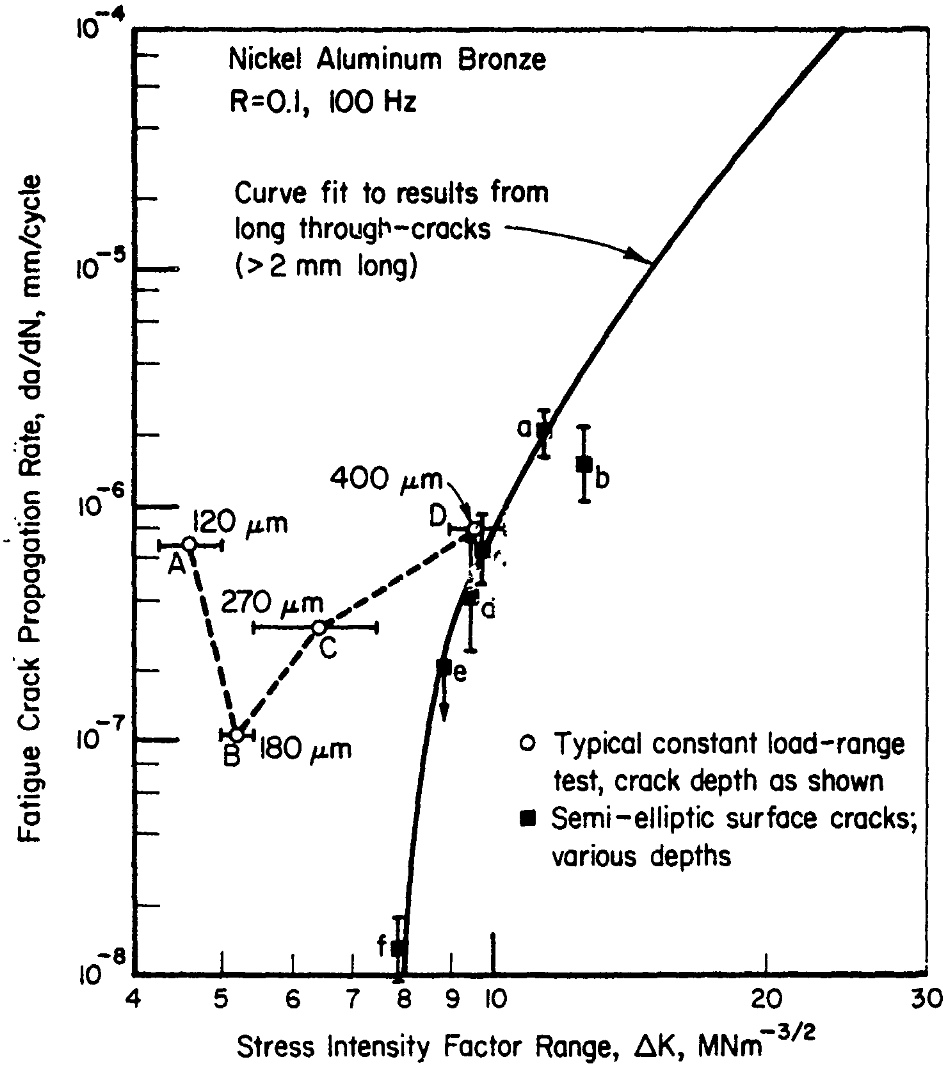

FIGURE 3.10. PROPAGATION RESULTS FROM ALUMINUM BRONZE (Taylor and Knott [154])

through several grains. Certainly, given the argument of similitude made earlier, differences in either crack mode or mechanism can alter growth rate. Likewise, they can change the local closure behavior thereby confounding identification of the local value of R.

The same two factors, mode and mechanism, can still play a role even if conditions of constraint at the tip are not the same. Yield stress changes with grain size, and yield strength and local constraint control the plastic zone size. Thus, one could also argue that differences in mode and mechanism are manifestations of changes in deformation behavior--that is mechanics controlled deformation behavior is the cause, and differences in mode and mechanism and growth rate are the effects. By some this may be viewed as a chicken-and-egg argument. But, establishing factors controlling short crack behavior does involve sorting out the cause and its effects. Unfortunately, fractographic and deformation response data have not yet been published which identify the cause and effect behavior for these data.

It is probably worth noting in regard to smooth specimens (i.e., no initial notch or crack*) that they do end up initiating and growing cracks. Put another way, although the stress intensity factor (range or maximum) is initially zero, a crack eventually develops and may grow. In this respect, the applicability of the concept of a threshold for smooth specimens is questionable. Equally questionable is the use of ΔK or K_{max} in a transition domain from a no-crack (and, therefore, no stress intensity) to a cracked configuration. Clearly, the "initiation" process (and the size and shape of the microcrack it produces) is a transient state culminating in a long crack steady state condition. Limited data indicate that some finite volume of surface damage may be involved in the rapid evolution via a crystallographic

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* In structural alloys inclusions, voids, etc., may be considered as microcracks that have an associated inherent stress intensity. However, ductile pure metals like OFHC copper involve only copper and grain boundaries. These materials have no microstress raisers that develop an inherent stress intensity. And they still "initiate" transgranular cracks.

"pop-in" or by striation like growth through heavily damaged surface material [92,189]. This suggests that naturally initiated cracks may develop differently than those which grow from artificial defects--i.e., their transient behaviors may be different. It also suggests that the transition to long crack conditions is associated with the evolution of a steady state crack tip condition. The latter is simply a reaffirmation of the requirements of similitude stated earlier.

3.3 Near Threshold Growth of Short Cracks at Notches

A great deal of data were developed for notched specimens in the middle to late 50's, which imply that geometrically small and large cracks will grow at different rates, (e.g., [4]). A small sample of these data, derived in the study of nonpropagating or intermittently propagating cracks, is shown in Figure 3.11. Frost, et al demonstrated in the data for sample MBJM F1 that small cracks develop at rates much greater than steady state rates at larger lengths (and therefore, larger ΔK). This same trend is evident in sample MBJM A7. However, after limited growth the cracking process in this sample became dormant. Their work also showed the significance of microstructure, and of crack branching, although suitable fracture mechanics analyses did not yet exist to indicate the significance of branching.

Hunter and Fricke [3] also discussed the growth of short cracks in aluminum alloys. Their work suggested that metallurgical features may have been involved with pinning crack tips. They coined the term "hesitation" in regard to the intermittent growth behavior of cracks which previously had continuously grown ten fold in length. Figure 3.12 is illustrative of this behavior.

From the early 60's until the early 70's, interest in the behavior of crack growth from notches focused on macrocrack behavior. In 1972, Broek [24], studying the concept of effective crack length--the length at which a crack no longer recognizes the effect of a hole--developed the results reproduced in Figure 3.13. It can be observed in that figure that smaller cracks appear to grow at a rate in excess of their long crack counterparts. This study did not address the threshold regime.

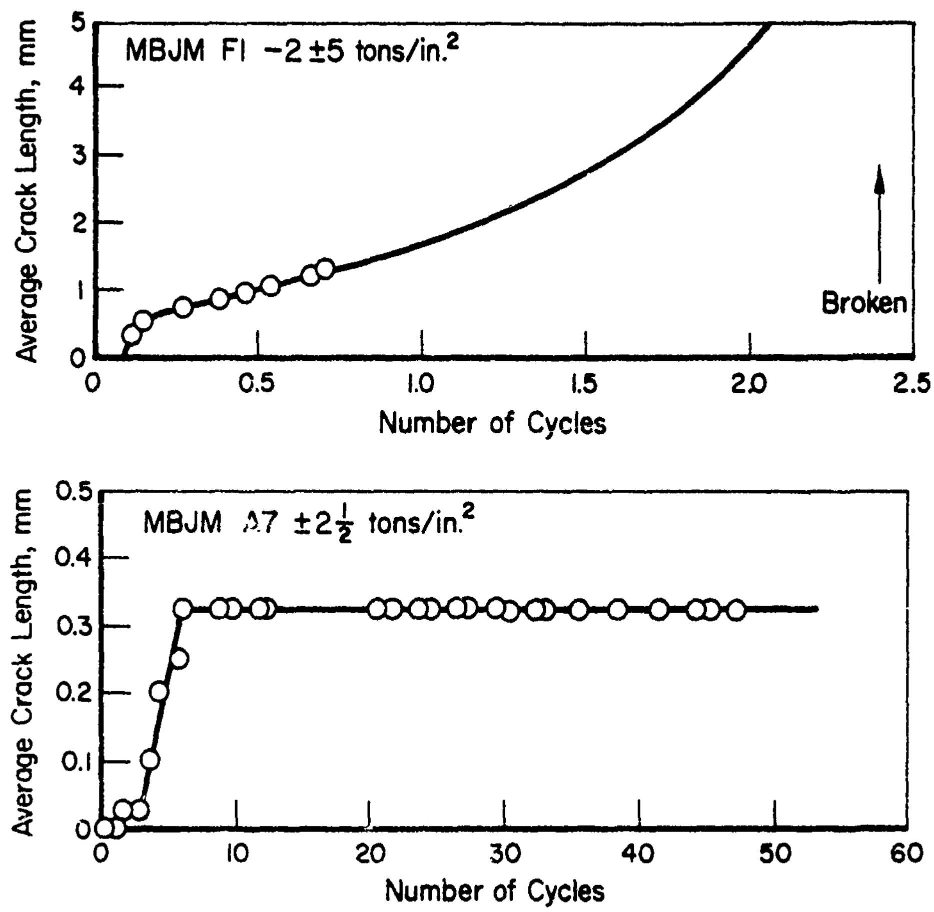

FIGURE 3.11. CRACK GROWTH VERSUS CYCLES FOR NOTCHED SAMPLES NEAR ENDURANCE (Frost and Dugdale [41])

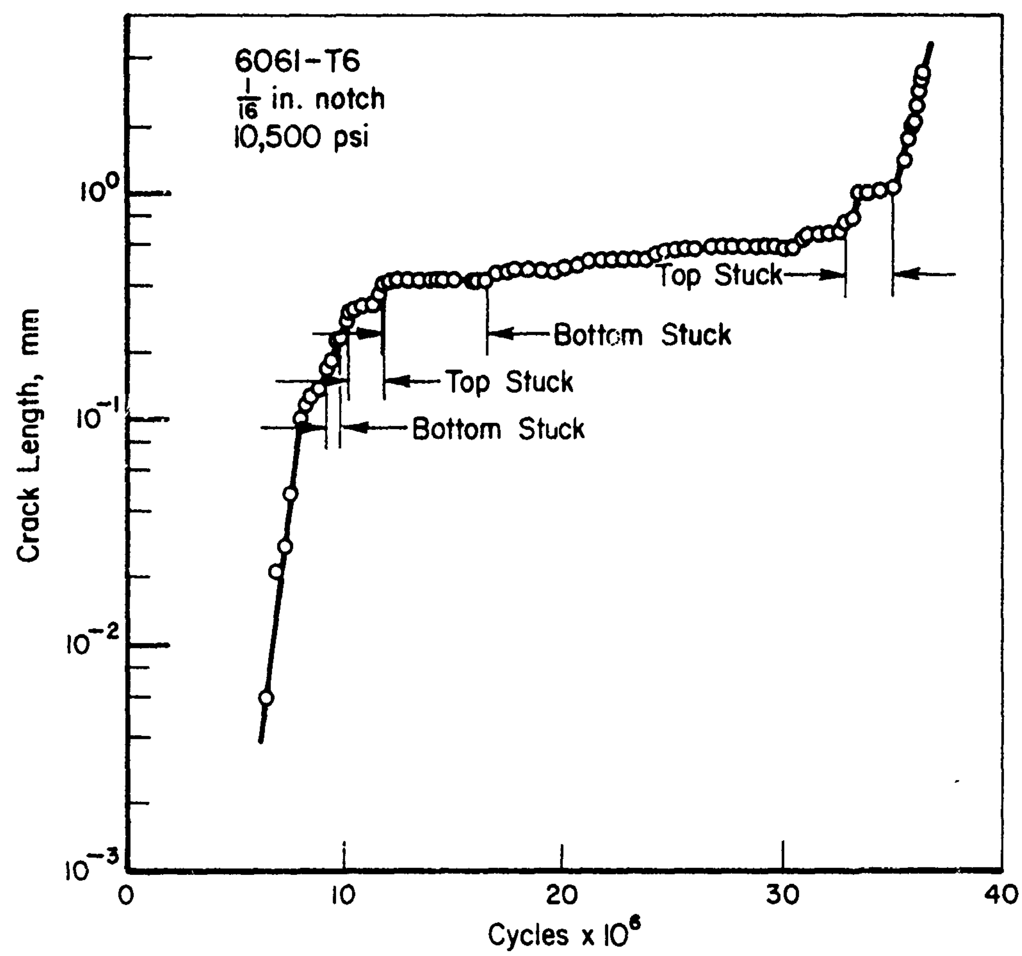

FIGURE 3.12. TYPICAL CRACK GROWTH CURVE WITH NOTATIONS OF MAJOR PERIODS DURING WHICH ONE END OF CRACK DID NOT GROW. THE LONG HORIZONTAL PORTION OF CURVE IS THE HESITATION PERIOD (Hunter and Fricke [3])

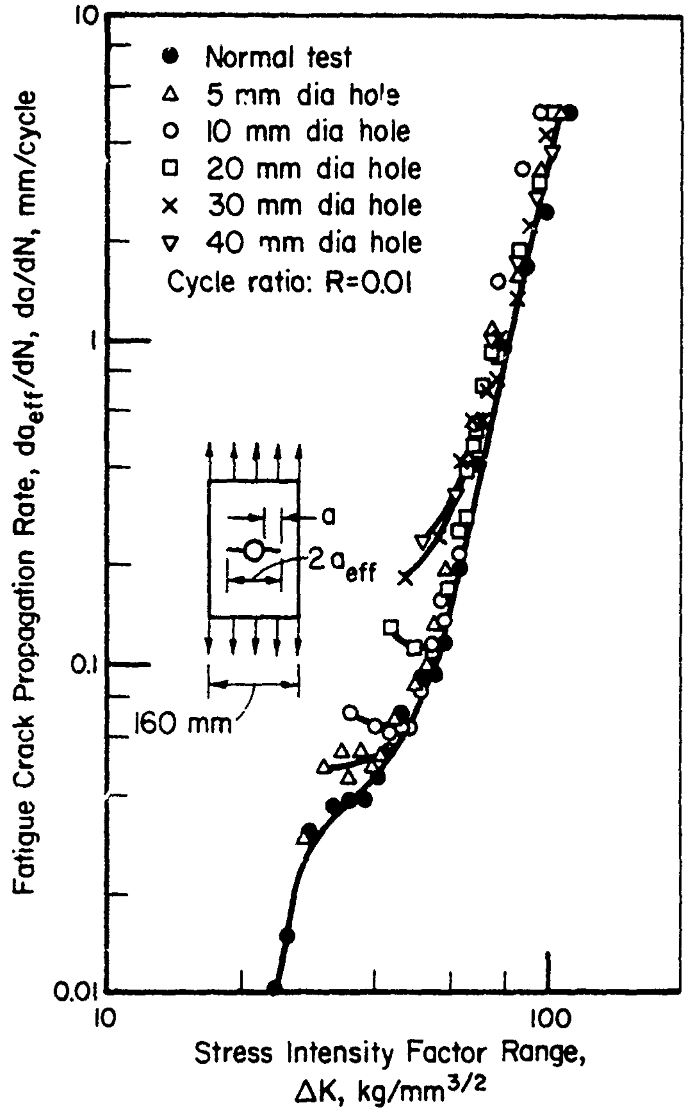

FIGURE 3.13. GROWTH RATE OF CRACKS AT HOLES. CENTRAL CRACK; SYMMETRIC CASE (Broek [24])

Data germain to the threshold condition for notched geometries have been generated by several investigators. These data suggest that sharply notched specimens give the same trends as smooth specimens do (e.g., [41]). This conclusion is true, however, only if the method they used to correlate data from smooth specimens with data from notched specimens is valid. It is argued by Cameron [173] that only smooth specimens should be used to develop data of the type plotted in Figure 3.2. Cameron's argument suggests that correlation of data from smooth and notched geometries is fortuitous. The limited data which bear on this issue suggest that his concern may be unfounded. However, if it is true that microcrack closure causes the difference between long and short cracks, its dependence on geometry would lend support to Cameron's point of view.

The results presented for the deeply notched specimens suggest that all of the near threshold observations made in regard to Figure 3.2 are also applicable here. One additional factor relates to the multiple initiation and branching of cracks in the notched speicmens. Both multiple initiation and branching indicate that the implicit assumption of a single plane fronted crack as used in developing these figures, is too simplistic. Poor correlation with LEFM is to be anticipated for them. For this reason, the observation that LEFM as applied in Figures 3.2, 3.3, and 3.10 cannot consolidate small and large cracks, does not indicate that limitations of LEFM are responsible. Before this could be concluded, it would also have to be shown that LEFM, properly applied, failed to consolidate the data. Multiple cracking and branching would make closure calculations difficult, so that accurate determination of ΔK^{eff} would be difficult. However, to be correctly applied, LEFM should use ΔK^{eff} and not ΔK. Thus, as difficult as they are, such calculations are the only way to conclusively evaluate the applicability of LEFM.

El Haddad et al [110] have also performed experiments which infer that LEFM will not uniquely consolidate near threshold growth behavior of small cracks at notches. Unlike other results, for example Tanaka et al [179], which involve measured growth rates, their results do not directly include such data. Instead, they involve a comparison of the observed and predicted values of ΔK_{th} (which includes a pseudo crack length instead of a physically measured one), with crack length. Because raw data are not presented, it is difficult to conclude which factors may give rise to the difference between short and long cracks based on LEFM. Furthermore, little is said regarding crack configuration, and fracture mode and mechanism. For this reason, this study cannot provide insight into what factors give rise to the small crack effect.

In summary, near threshold growth of short cracks at a sharp notch follows the same trends as presented and discussed for smooth specimens. This can be anticipated because in these two situations similar factors control growth. Multiple initiation and crack branching appear to be major factors for specimens with naturally initiating cracks. Little data exist for bluntly notched geometries under true threshold conditions making it difficult to draw conclusions for that situation.

3.4 Finite Growth Rate Behavior of Short Cracks in Notched Samples

A number of experimental studies have pursued the behavior of short cracks at notches since Broek's work [24]. Pearson [36] has examined the behavior of short cracks at notches in an aluminum alloy. His test conditions involved some local inelastic action due to notch yielding. A growth rate trend similar to that of Broek was observed. Hill and Boutle [80] studied a similar situation in various steels and noted that LEFM did not uniquely consolidate short crack growth rate behavior. They ascribed this to inelastic action at the notch, but did not elaborate further. Gowda, et al [25] developed data which exhibit very large initial growth rates. Following the work of Neuber for cracks, they modified the Bowie solution [2] to account for inelastic action, but found that the relationship between stress-intensity and rate no longer followed the usual power law behavior.

Following these initial studies a number of other situations have been explored. El Haddad, et al have developed data for a mild steel [94,111]. Their results are reproduced in Figure 3.14. They reflect situations of very contained notch plasticity, and indicate LEFM underestimates growth rate based on long crack data. Leis, et al [178,189,200] have explored a mild steel, a rail steel, and two high strength aluminum alloys. They also found that a

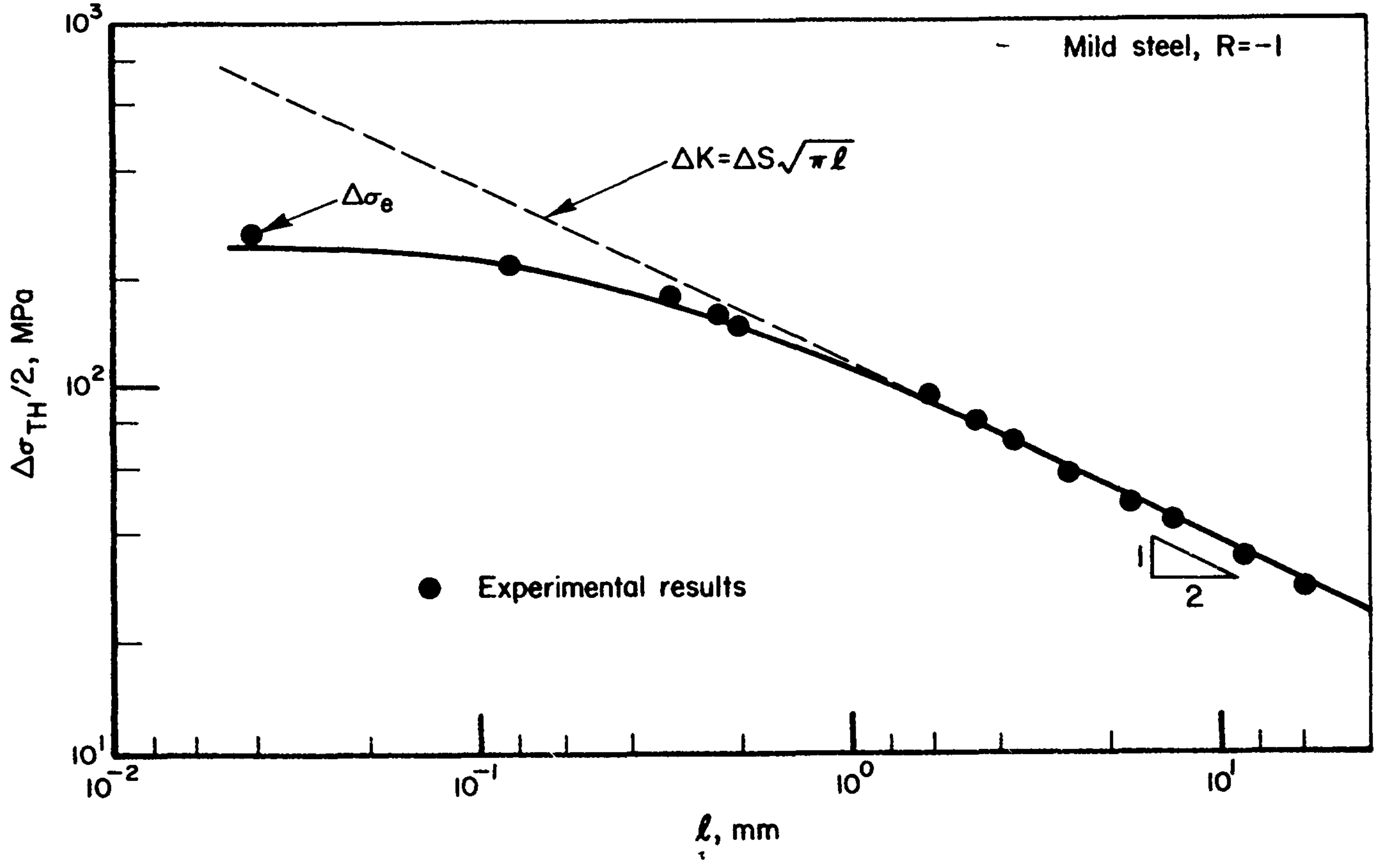

FIGURE 3.14. EFFECT OF CRACK LENGTH OF THRESHOLD STRESS AMPLITUDE FOR FATIGUE CRACK GROWTH IN G40.11 STEEL (El Haddad, et al [94,111])

single parameter LEFM analysis underestimates growth rates in situations that range from confined to unconfined notch flow and from corner to plane fronted cracking.

In the work of El Haddad, et al, it is suggested that LEFM fails to consolidate the growth of small cracks at notches because of inelastic notch root behavior and differences in constraint that are discussed in terms of grain size. These studies, however, say little concerning the nature of the initiation process, nor do they discuss details of crack configuration and fracture morphology. Based on visual observations with the unaided eye after failure, El Haddad has noted, in a private communication, that all cracks are plane fronted at all lengths. Unfortunately such observations cannot establish details of the microcracking process.

The experiments of Leis, et al indicate that the transition from short to long crack growth behavior correlates with the plastic zone size of the notch root. These results are shown in Figure 3.15. As shown in Figure 3.16 they argue that the plastic zone which contains cracks during their early growth is displacement controlled so that simple LEFM analysis based on nominal R values is inappropriate [178,205]. They also show that corner crack behavior in its transition to a plane fronted crack correlates with observed anomalous growth rate trends [189]. Only after one of several cracks have initiated and grown to lengths approaching 100 μm, does one crack selectively grow to failure. This study indicates that initiation is often crystallographic, involving an apparent pop-in behavior. Initiation transients from pop-in are argued as being a source for the high initial growth rate behavior of small cracks.

In general, all of the above studies suggest that simplified LEFM as applied by the various investigators fails to consolidate growth behavior of small cracks from notches. They show that even after LEFM is corrected to account for corner cracking the anomalous growth rate behavior remains. They indicate that multiple initiation develops, but is not accounted for in ΔK calculations. Furthermore, local closure that may develop for small cracks is not accounted for. This is because either ΔK or ΔK_{max}, used as a basis for

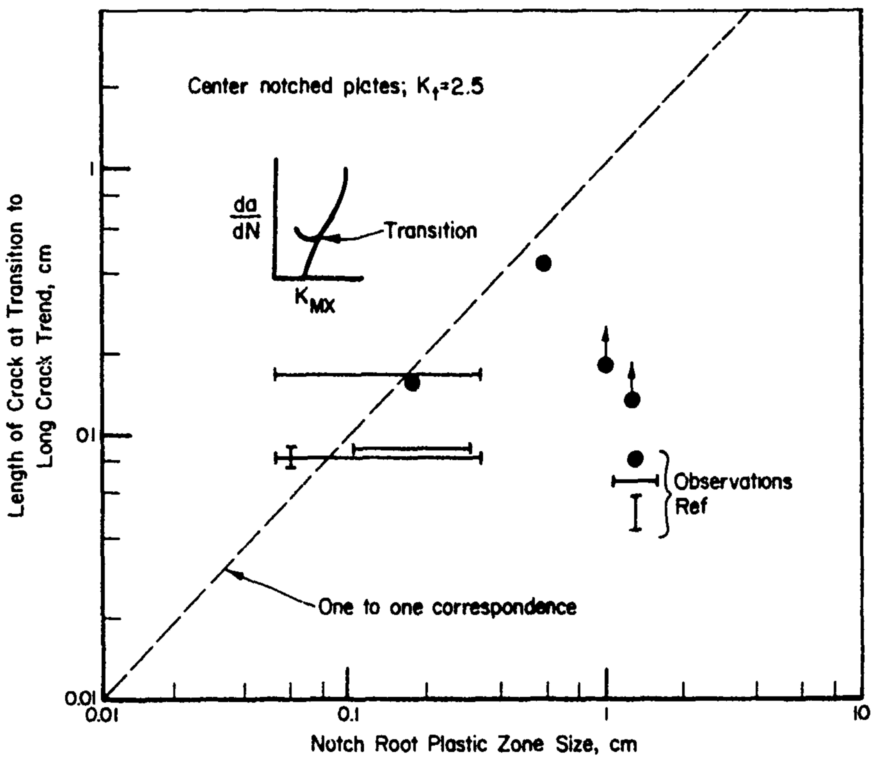

FIGURE 3.15. CORRESPONDENCE BETWEEN TRANSITION CRACK LENGTH AND NOTCH ROOT PLASTIC ZONE SIZE (Leis [200])

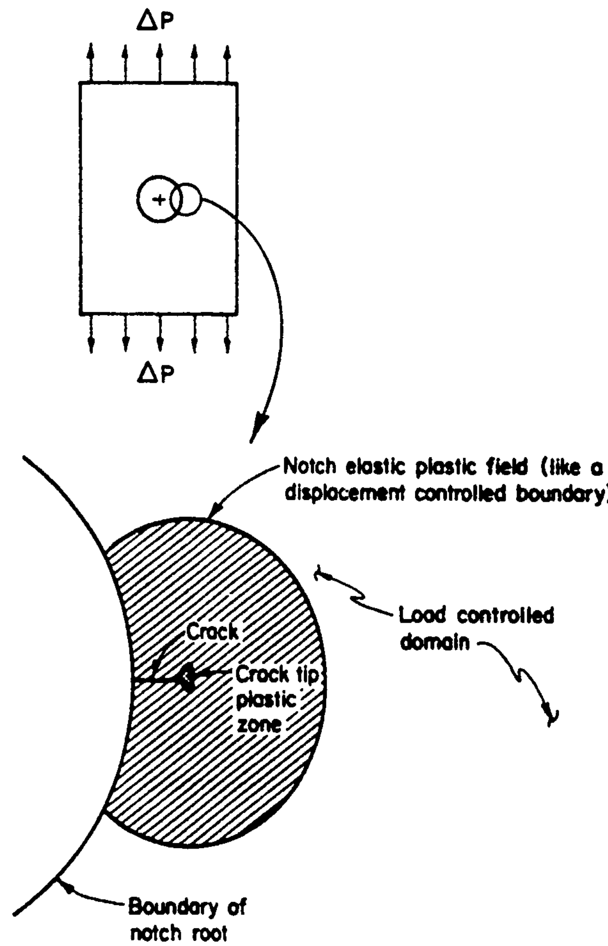

FIGURE 3.16. DISPLACEMENT CONTROL OF CRACKS CONTAINED WITHIN NOTCH PLASTIC ZONES (Leis, et al [178,189,200])

consolidation, is a far field parameter which is insensitive to notch root and local crack tip inelastic action.*

As a rule, the data considered suggest that the anomalous short crack behavior at notches may be much different than for smooth specimens if notch root inelastic action is involved.** LEFM as applied does not seem to consolidate growth behavior, but none of the data conclusively prove that LEFM cannot be successfully applied in a context that accounts for notch root inelastic action.

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* Heuristic arguments imply that (multiple) short cracks loaded in displacement control and developed under fully reversed loading would have a very low opening load. As noted in [205], LEFM analyses can be shown to consolidate crack growth from notches provided that the inelastic action in the notch field and local closure are accounted for.

** Under circumstances of $R > 0$, inelastic action may occur primarily on the first cycle. Even though subsequent deformations may be largely elastic, the far field stress still acts through the displacement controlled condition created in the first cycle for cracks contained in that field.

4. CORRELATIVE MODELS

A general observation is that correlative models tend to be influenced by the character of the data generated. Investigators appear to try to identify what they believe to be responsible for the breakdown in an LEFM analysis of their data, and then try to develop a model to correct for it. Correlative models also evolve from a consideration for the limitations inherent in LEFM. Some correlative models involve empirically derived fitting constants--others do not. In this section, a review and analysis of the major correlative models is made in chronological order. Purely mechanics based models will be dealt with in the next section. Smooth and notched geometries tend to be generically different situations and are discussed separately.

4.1 Correlative Models for Smooth Specimens

One of the earliest correlative models (Circa 1971) is due to Frost, Pook, and Denton [20]. They reanalyzed the data developed earlier by Frost, et al and noted that thresholds for growth of short cracks are less than their long crack counterparts. They then suggested a purely empirical model that was a function of the alternating stress, ΔS, and the crack length, a.

The next correlative model, developed by Ohuchida, et al, [41] (Circa 1975), built on the insight gained by Frost, et al [4] and on their own earlier experiments [28]. They had shown that the cyclic plastic zone size and cyclic crack opening displacement correlated with growth rate. These authors were the first to plot stress and crack length, as shown earlier in Figure 3.2. They defined equivalent crack length, a_e, as

$$a_e = \frac{\Delta K^2}{\pi \Delta S^2} , \tag{4.1}$$

which can be interpreted as crack length normalized to that in an infinite sheet. Further they defined a stress intensity factor range

$$\Delta K = \Delta S \sqrt{\pi (a_e + r_p)} \; f(\tfrac{a}{w}) \tag{4.2}$$

in which r_p is defined by analogy to the Dugdale model [9] and therefore is a function of applied stress. A geometric function, $f(^a/w)$, was implicit in their derivation and to this end has been added explicitly to Equation (4.2).

Using this correlative framework, Ohuchida, et al showed that the data trend evident in Figure 3.2 can be predicted. An example taken from their paper is shown in Figure 4.1. Further, they showed that the value of r_p decreases as a power law function of the material yield strength, as shown in Figure 4.2.† Finally,* they show that the equivalent crack length for smooth specimens, a_o, is numerically equal to r_p, and is on the order of the grain size. But, as shown in Figure 4.3, their data suggest a limit below which this 1-to-1 correspondence breaks down. They suggest that this breakdown is associated with a change from reversed slip initiation behavior to an inclusion initiation mechanism. This suggests that the relationship shown in Figure 4.2 will break down with the change in initiation mode.

Two years later, the frame work advanced by Ohuchida, et al was investigated further. Smith [61], in a paper concerned with physically small cracks also suggested that a form similar to Equation (4.1) might be appropriate. At this same time, Staehle [75] noted that smooth specimen behavior provided an upper bound to the stress that would grow a crack of a given length. As noted in Figure 4.4, reproduced from that paper, this upper bound lies well below the behavior associated with LEFM. Apparently independent of Staehle, Smith published his results in a note as a current research report. Based on exactly the same rationale as that advanced by Staehle, Smith suggested that threshold data for long and short cracks would yield a plot like Ohuchida's that is shown in Figure 4.1. Further, he argued that a crack size of 0.25 mm is a lower bound for the proper application of LEFM concepts.

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† Given that strength is inversely proportional to the square root of grain size, Figure 4.2 infers that r_p is also a power law function of grain size.

* They also deal with notches, however, a discussion of this portion of their work will be deferred until the section on notched specimens.

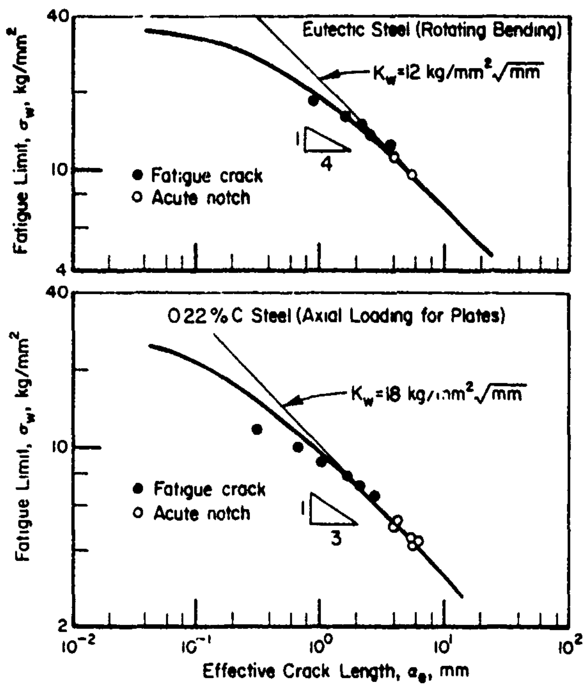

FIGURE 4.1. CORRELATION OF SHORT CRACK GROWTH RATE BEHAVIOR USING A CRACK TIP PLASTICITY ADJUSTMENT (Ohuchida, et al [41])

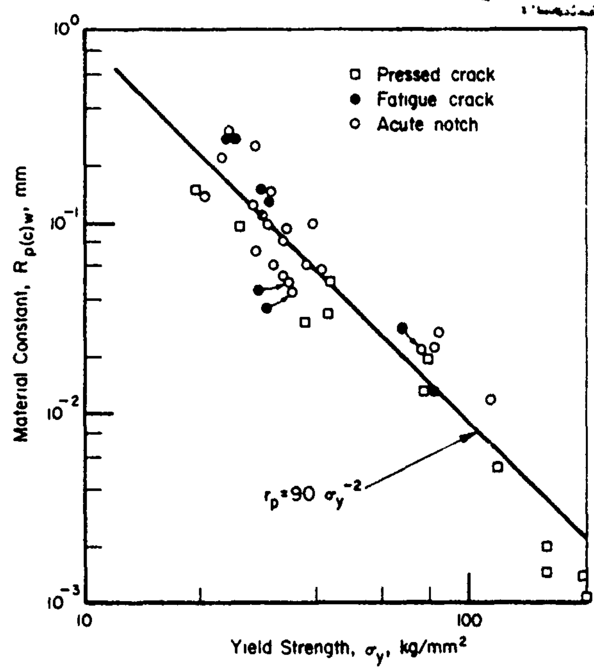

FIGURE 4.2. DEPENDENCE OF THE CORRELATING PLASTIC ZONE ON MATERIAL YIELD STRENGTH (Ohuchida, et al [41])

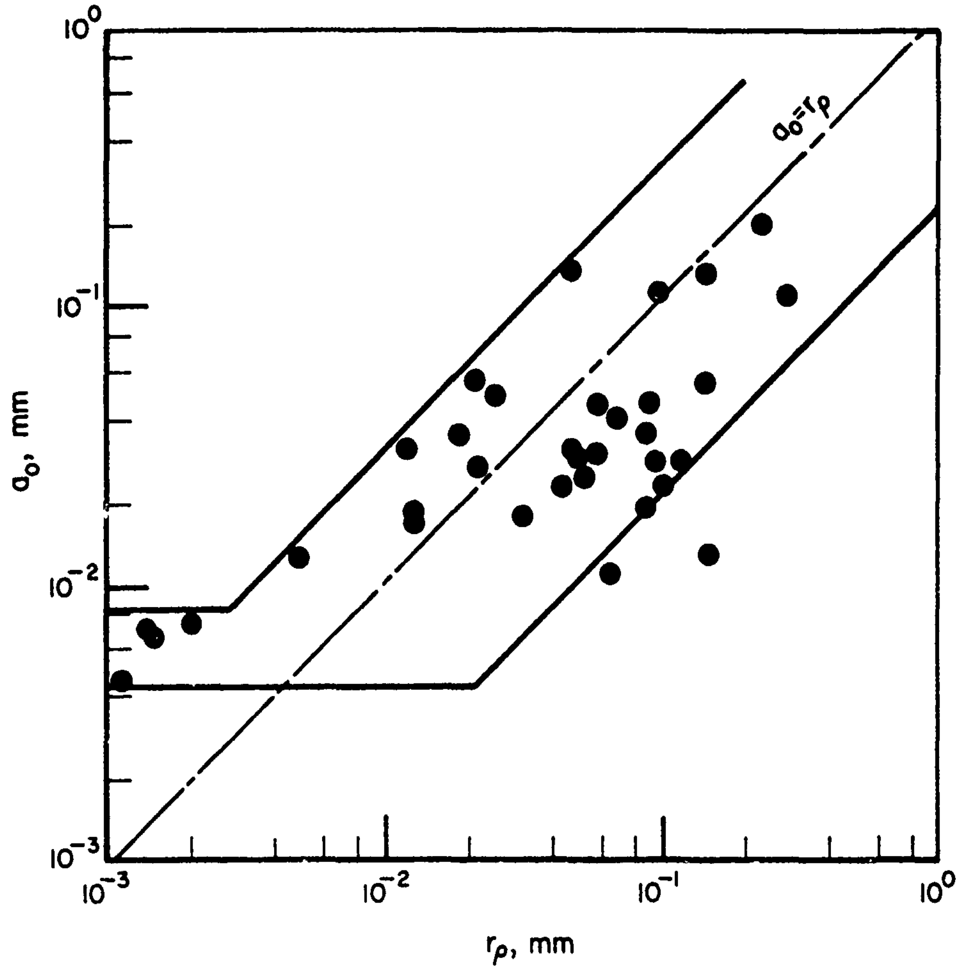

FIGURE 4.3. DEPENDENCE OF THE CORRELATING PLASTIC ZONE ON THE CRACK LENGTH IN SMOOTH SPECIMENS (Ohuchida, et al [41])

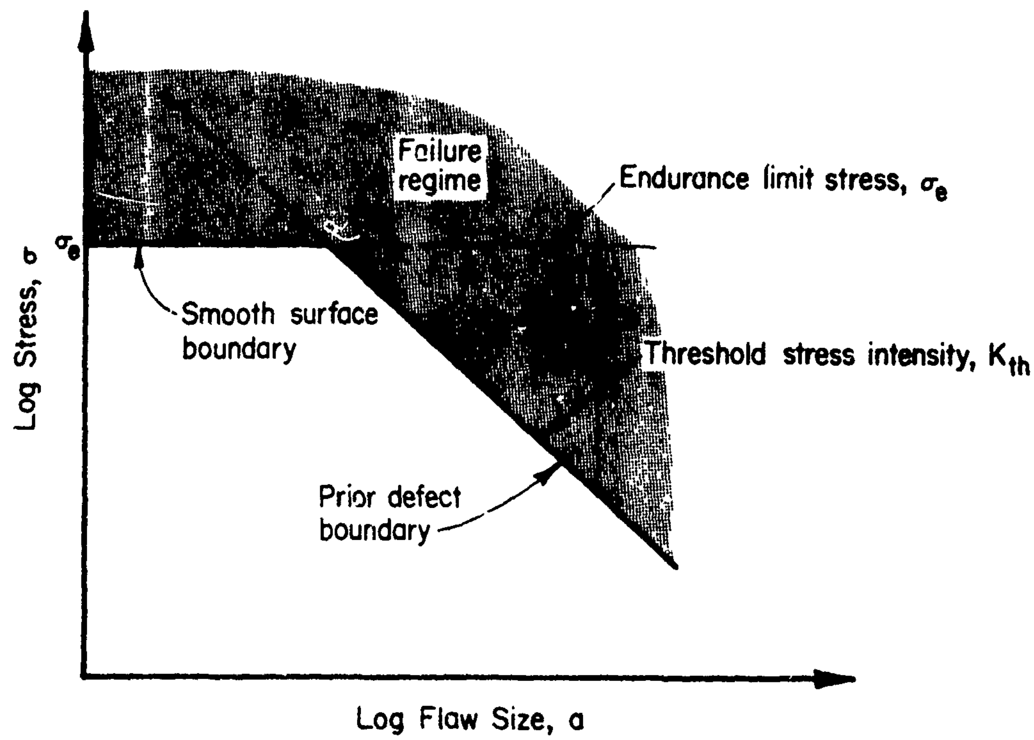

FIGURE 4.4. FAILURE BOUNDARY DEFINED IN THE FRAMEWORK OF STRESS INTENSITY AND FATIGUE LIMIT FOR THE TWO CASES OF SMOOTH AND DEFECTED SURFACES (Staehle [75])

Cracks larger than 0.25 mm would give results which correlate using LEFM while cracks smaller than 0.25 mm should be correlated using the endurance limit. Smith arrived at the boundary crack size through a heuristic calculation which involved either an inadvertent arithmetic error or a typographical error. His calculation of $50 \times 5 \times 10^{-4}$ mm incorrectly gave 0.25 mm, whereas the correct result is 0.025 mm. This would suggest that LEFM should be applicable to most of the cracking discussed earlier in the Phenomenological Section, since 25 μm is on the order of the lower bound crack size discussed.* With regard to Figure 4.5, observe that indeed as the crack size decreases there is a length below which ΔK_{th} is no longer constant, supporting the views of Staehle and Smith. However, it should be emphasized that Figure 4.5 is merely another way of plotting the same information shown in Figure 4.1, first advanced by Ohuchida. Nothing fundamentally new in concept or insight is gained from the use of Figure 4.5 as compared to Figure 4.1.

In 1978 and 1979, additional papers appeared which pursued the theme established in Figure 4.1. Kitagawa, et al [105] argued the significance of a total strain intensity factor, ΔK_ε. They define ΔK_ε as

$$\Delta K_\varepsilon = \Delta_\varepsilon^t \sqrt{\pi a} \quad f\left(\frac{a}{W}\right)^{\dagger} \tag{4.3}$$

where Δ_ε^t is the total bulk strain range. They showed that a parameter of this form achieved limited consolidation of short crack growth behavior for several soft steels, but noted that consolidation could not be achieved using the

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* Staehle's plot suggests that the transition crack size, a_t, between endurance limit data, $\Delta\sigma_e$, and LEFM threshold, ΔK_{th}, can be established empirically from these two parameters. Numerically, this would be equivalent to solving $\Delta K_{th} = \Delta\sigma_e \sqrt{\pi a_t}\ f(a/w)$ for a_t. This would produce values in the range 1 μm < a_t < 500 μm for most of the data available, a range which contains the heuristic result of Smith.

† Equations of this form have been suggested over the years for various purposes, (e.g., [95]), including applications to relatively short cracks growing from notches.

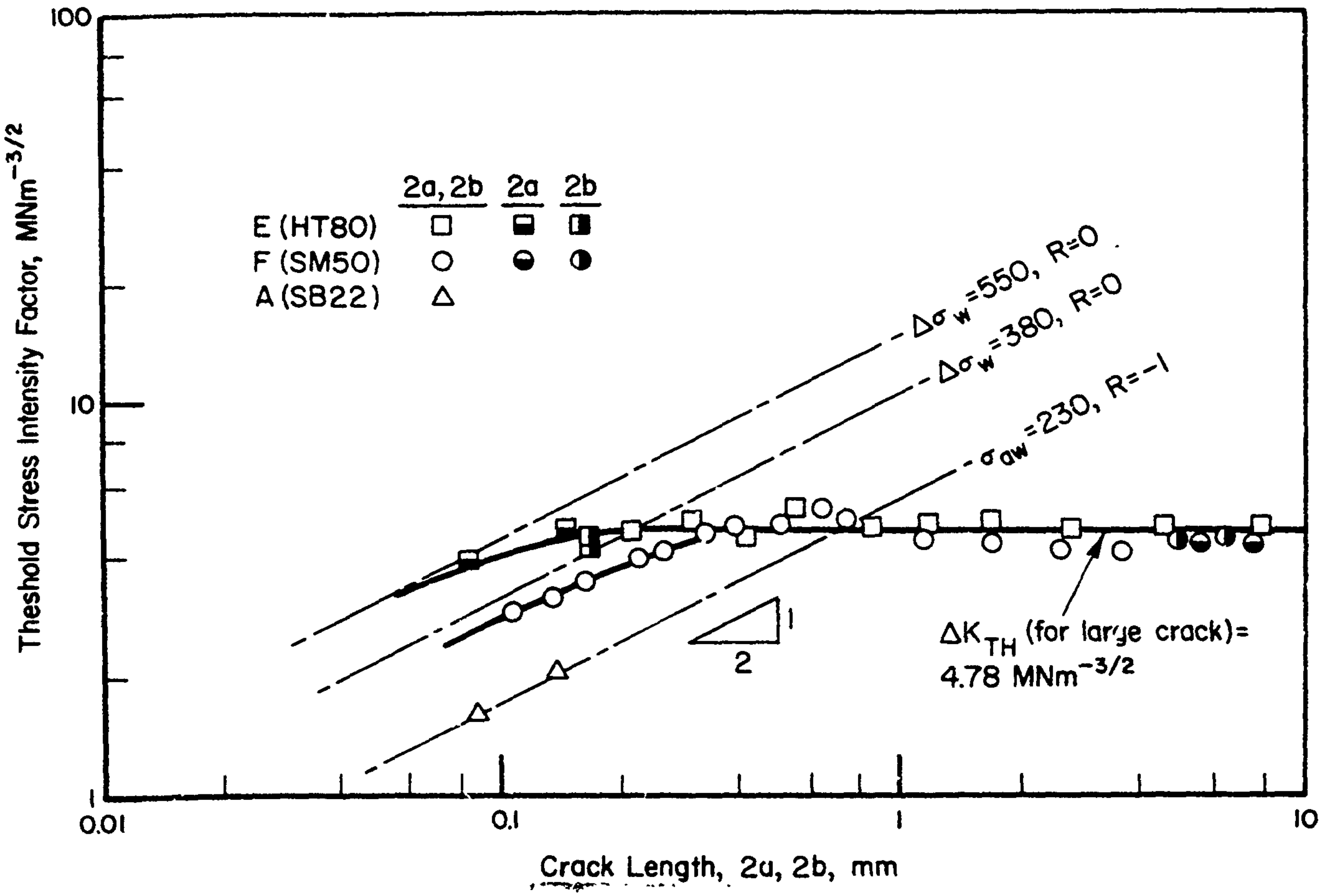

FIGURE 4.5. BREAKDOWN OF LEFM AS A FUNCTION OF SURFACE CRACK LENGTH AND DEPTH (Kitagawa, et al [105])

plastic strain component. El Haddad et al [111] suggested an empirical model to characterize the trend of Figure 4.1. They defined

$$K = \Delta S \sqrt{\pi(a + \ell_o)} \quad f(\tfrac{a}{W}) \quad , \tag{4.4}$$

a form of which is mathematically identical to Equation (4.2). Here ℓ_o is an empirical parameter evaluated by comparing threshold behaviors for long and short cracks:

$$\ell_o = \left[\frac{\Delta K_{th}}{\Delta\sigma_e}\right]^2 \frac{1}{\pi} \quad . \tag{4.5}$$

In Equation (4.5) ℓ_o would be interpreted in an LEFM sense as the length of crack which initiates but does not propagate at the endurance limit, $\Delta\sigma_e$.* Recall that for smooth specimens, $\Delta K_e = \Delta\sigma_e \sqrt{\pi\ell_o}$. To be precise, Equation (4.5) should include a geometric correction factor, otherwise the limiting value of ℓ_o is geometry dependent. Furthermore, as defined by Equation (4.5), the value of ℓ_o depends on stress ratio which may be different for both ΔK_{th} and $\Delta\sigma_e$. Finally, closure is a factor which can confound the use of ΔK. Even if nominal values of R are identical, closure appears to be a function of crack length and tends to depend on geometry. Thus, ℓ_o may be useful for correlating data from comparable geometries, but its practical significance remains an open question.

Several interpretations of the physical meaning of ℓ_o are advanced by El Haddad, et al in [111]. Initially, ℓ_o is defined as an empirical parameter. They then note that it may reflect the reduced flow resistance of metal at a free surface. It is additionally noted that ℓ_o may reflect the fact that for short cracks the singular part of the stress field embodied in K may be inadequate, as suggested by Talug and Reifsnider [62]. Data later developed by El Haddad, et al [174] suggest that the value of ℓ_o may

*As discussed earlier, there is no fundamental basis for comparing endurance limit and threshold concepts as is done in Equation (4.5). The results of such comparisons do embody the factors which cause differences between long and short cracks in an empirical fashion. However, because these factors tend to be geometry dependent, the requirements of similitude may not be satisfied in applications to other geometries.

not depend consistently on strength. It is unlikely that this inconsistency is due to the failure of the singular term to characterize the near tip field for short cracks. It seems more probable that unconstrained surface flow and its influence on local closure would explain this. Or, given the analogous form of Equation (4.4), it may also be due to a change in the initiation mechanism as strength increases (cf Figure 4.3). Even with its apparent physical incompatibilities, the simple nature and the utility of the concept have led El Haddad, et al to publish numerous papers demonstrating its successes, but only for a limited range of experimental results. The fact remains that ΔK_{th} and $\Delta\sigma_e$ are philosophically incompatible concepts. As crack length goes to zero, $\Delta\sigma_e$ must approach infinity--therefore, the LEFM K concept is no longer valid.

In 1979 Usami and Shida [109] published some work which was tied to the 1975 work of Ohuchida, et al. They provided an extensive demonstration that the concept advanced by Ohuchida, et al correlated short crack behavior.

The year 1981 brought a reemphasis of the earlier thoughts of Staehle [75] and Smith [61]. Recall that both had postulated two bounds to the trend curve of Figure 4.1. Smith argued that below a length 50 x 5 x 10^{-4} mm, the endurance limit was appropriate whereas above that length, the use of LEFM was appropriate. In 1981, Cameron [173] refined the earlier proposal of Smith to include a smooth transition in a region of crack lengths between these limiting conditions which embraced El Haddad's value of ℓ_0 as well as much of the range of values of a_t shown in Figure 4.6. The lower bound length, ℓ_1 is interpreted as a measure of the weakening defects in the material, or as the influence of free surface on the flow process and microcracking. However, in view of the breakpoint in Figure 4.3, the value of ℓ_0 may depend on the mode of initiation. It may also depend on whether or not cracks bifurcate, which in turn appears to depend on grain size [180]. The length ℓ_2 is argued as the length beyond which microstructure has no effect on conventional fracture mechanics can be employed.

Cameron [173] states that ℓ_1 and ℓ_2 will depend on R which is equivalent to admitting macroscopic closure as an important factor. Their interpretation of ℓ_2 as being dependent on microstructure precludes the influence of microclosure for decreasingly smaller cracks. But since ℓ_1

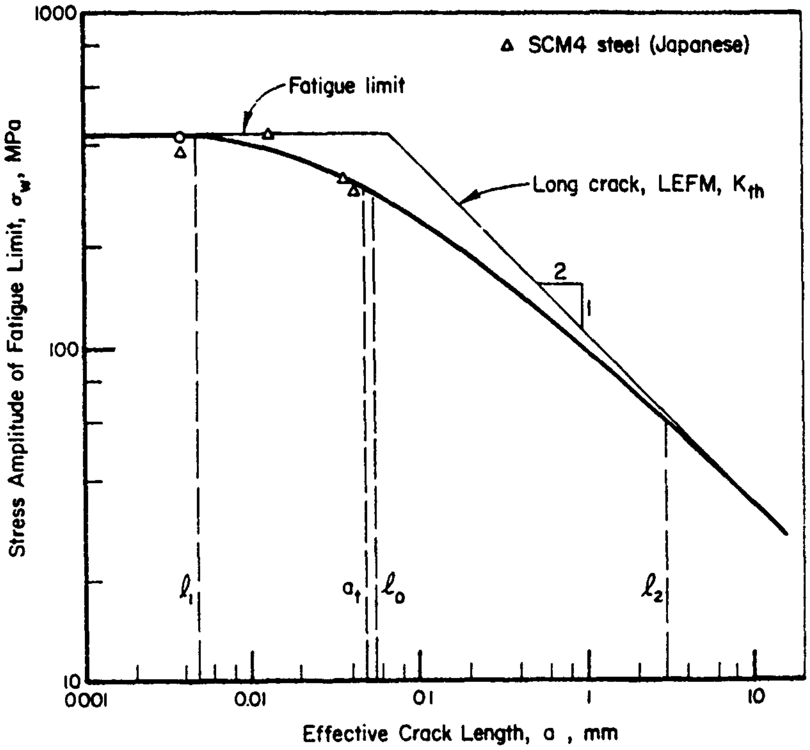

FIGURE 4.6. DIMENSIONS ℓ_1, a_t, ℓ_0, and ℓ_2 ILLUSTRATED USING DATA DEVELOPED BY USAMI AND SHIDA [109]

and ℓ_2 are empirically defined* their physical interpretation is in no way related to the ability of their model to consolidate their own data. It is, however, a factor in any attempt to apply the values of ℓ_1 and ℓ_2 to situations where crack length dependent microscopic and macroscopic closure differ from the geometry that empirically defines the values of ℓ_1 and ℓ_2. Likewise, if the modes of fracture change with crack length or stress level, our earlier arguments about similitude suggest that the values of ℓ_1 and ℓ_2 are of limited practical utility.

Morris, et al began to publish papers in 1979 that presented research progress toward an empirical model of the microcrack growth process [108,131,141]. Essentially their model is an empirical combination of ΔCTOD (more correctly ΔSCOD) as the driving force modified by a closure component. Thus, the essence of their model is an effective range of SCOD, consistent with the 1976 suggestion of Seika [55]. Earlier work presents the model in the absence of correlation with growth rate. Later work indicates successful correlation, although, as far as we can tell, the model has not been applied beyond data inherent in its formulation.

For models based on CTOD, the growth per cycle will be some function of CTOD only so long as growth is due to kinematically irreversible slip at the crack tip. Whether or not the amount of advance will relate uniquely to some ΔCTOD, independent of the stress and instantaneous crack length, remains an open question. For example, opening or blunting action at a crack tip that involves many slip planes at high stresses may generate many slip planes in back of the previous tip. That slip as well as the slip involved in advancing the tip, contribute to the total CTOD. Thus, a portion of the CTOD may not be involved with advance. What is still not clear is whether or not differences in crack length and constraint at small cracks will cause this portion of the ΔCTOD to depend on crack length and geometry. It is clear that ΔCTOD will depend on crystallographic orientation. In this respect the value of ΔCTOD will depend on the length of the crack front. Of course the crack closure will also depend on length. Thus, the interpretation of ΔCTOD should truly be

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*The parameters ℓ_1 and ℓ_2, as well as a_t are empirical in the same way that ℓ_0 is empirical.

one of crack tip opening. In this respect, the work of Nisitani and Kage [81] suggests that the closure effect is a very near tip phenomenon. Thus, CTOD must also be evaluated near tip (~ within 10 μm for the case studied in [81]).

4.2 Correlative Models for Notched Specimens

One group of researchers developed correlative models for notched specimens in a manner analogous to the way they developed models for smooth specimens. These include, in chronological order, the work of Ohuchida, et al [41], Usami and Shida [109], and El Haddad, et al [111].

In the case of Ohuchida, et al, the crack tip plastic zone was again considered to be the controlling parameter. They evaluated r_p for use in Equation (4.2) empirically as a function of notch root radius to establish a criterion for initiation near endurance conditions at notches. The correlation established was not thought to depend on stress. (Perhaps this worked because the endurance stress level was similar for the materials studied.) Furthermore, because endurance conditions were examined, the crack tip plastic zone was not dominated by that of the notch root. They also established the value of r_p for use in Equation (4.2) that was relevant for growth, through consideration of the notch stress field. Significantly, the value of r_p established for endurance was a constant, whereas that established for propagating cracks was a function of the crack length and the ratio of the endurance stress to the yield stress. The model of Usami and Shida is so similar to that of Ohuchida, et al, that they can be considered one and the same. In contrast to Ohuchida, et al and Usami and Shida, who consider r_p to depend on geometry, El Haddad, et al use the value of ℓ_0 determined from smooth specimens to correlate notched specimen behavior. Because of the manner in which these models are formulated, that due to Ohuchida, et al or the similar model discussed by Usami and Shida are more general, and offer the greatest potential for successful application to a wide range of data.

Both Ohuchida, et al and El Haddad, et al have developed models that result in a constant value of r_p (or ℓ_0) for a given material and specimen geometry. Applications of these models (with constant values of their empirical parameters) have only dealt with situations where very confined flow develops at notch roots. However, recent analysis [190] has shown that a constant value of such a parameter does not consolidate crack growth in the inelastic gradient field of a notch. As shown in Figure 4.7, significant error remains. In this case, the ℓ_0 concept was used. These data also suggest that cracks grown in an inelastic notch root field do so under displacement control of the elastic plastic boundary. With reference to Figure 3.16, note that for cracks within the inelastic field of the notch, the crack grows in a plastic zone other than that due to the crack. Care must be taken to ensure that the influence of the notch field is accounted for before LEFM can be expected to apply. To this end it has been postulated [200,205] that in inelastic notch fields cracking can be correlated using

$$K_{mx} = 1.12\ \varepsilon_{mx}\ E_m\ \sqrt{\pi a} \tag{4.6}$$

where ε_{mx} is the cyclically stable maximum strain in the material element at the depth of interest (in the absence of the crack), E_m is the monotonic modulus, and a is the length of the crack measured from the notch root. The product of ε_{mx} and E_m is used to estimate a pseudostress in keeping with the linear elastic nature of LEFM. An application of Equation (4.6) to rail steels has resulted in accurate prediction of the cracking behavior for cracks as small as 50 μm [205].

If crack growth in an inelastic notch field is controlled by that notch field and not the far field parameters as indicated in Figure 3.16, then the transition to long crack behavior should occur when the crack approaches the inelastic-elastic boundard. This behavior is shown in Figure 4.8. The limit of the LEFM notch field, for circular notches, based on an analysis by Novak and Barsom [47] and later by Smith and Miller [66,74] is also shown on this figure.* These LEFM "equivalent" crack lengths suggest that the crack should behave in accordance with an LEFM analysis for a long crack in a center cracked panel. Some cracks are observed to be within the LEFM definition of the notch field; others are well beyond it. In this respect, LEFM analysis

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* Note that the form of the relationship developed first by Novak and Barson and then by Smith and Miller is identical. They differ only in a constant whose value is chosen somewhat subjectively.

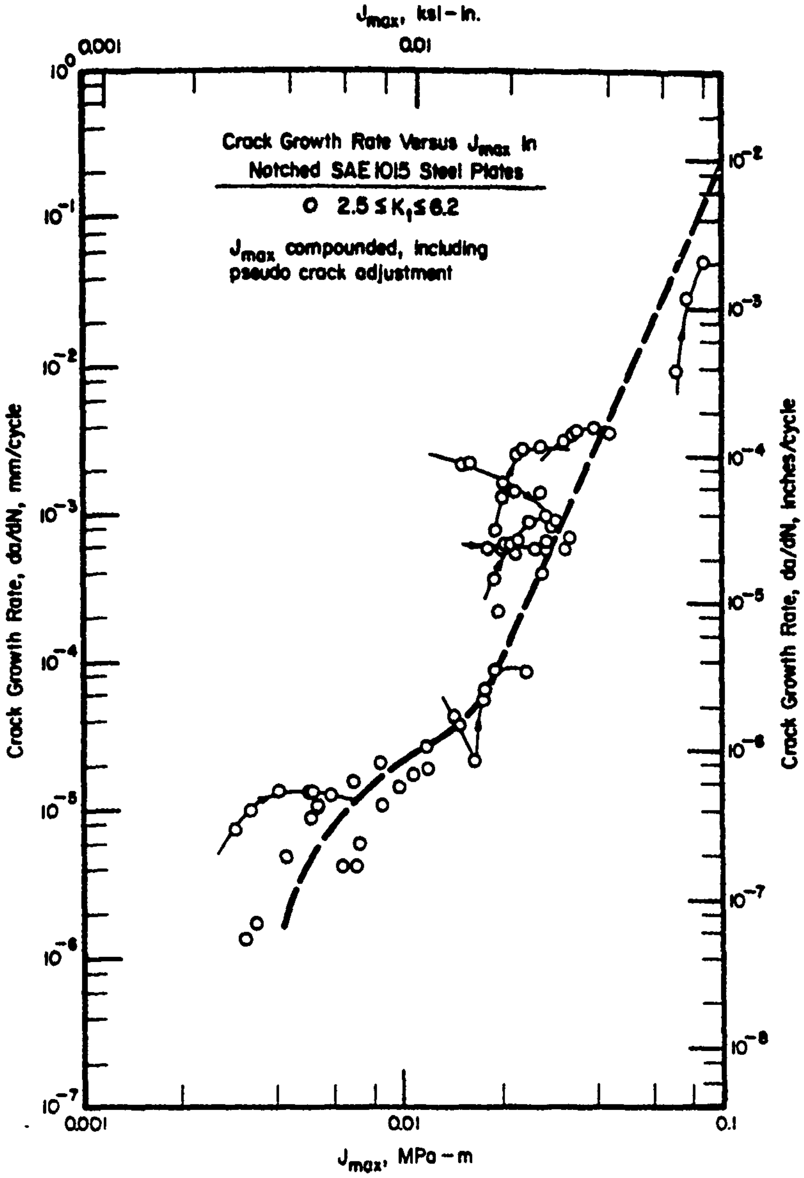

FIGURE 4.7. CRACK GROWTH RATE VERSUS J_{MAX} FOR SHORT AND LONG CRACKS INCLUDING A PSEUDO CRACK LENGTH ADJUSTMENT (Leis [190])

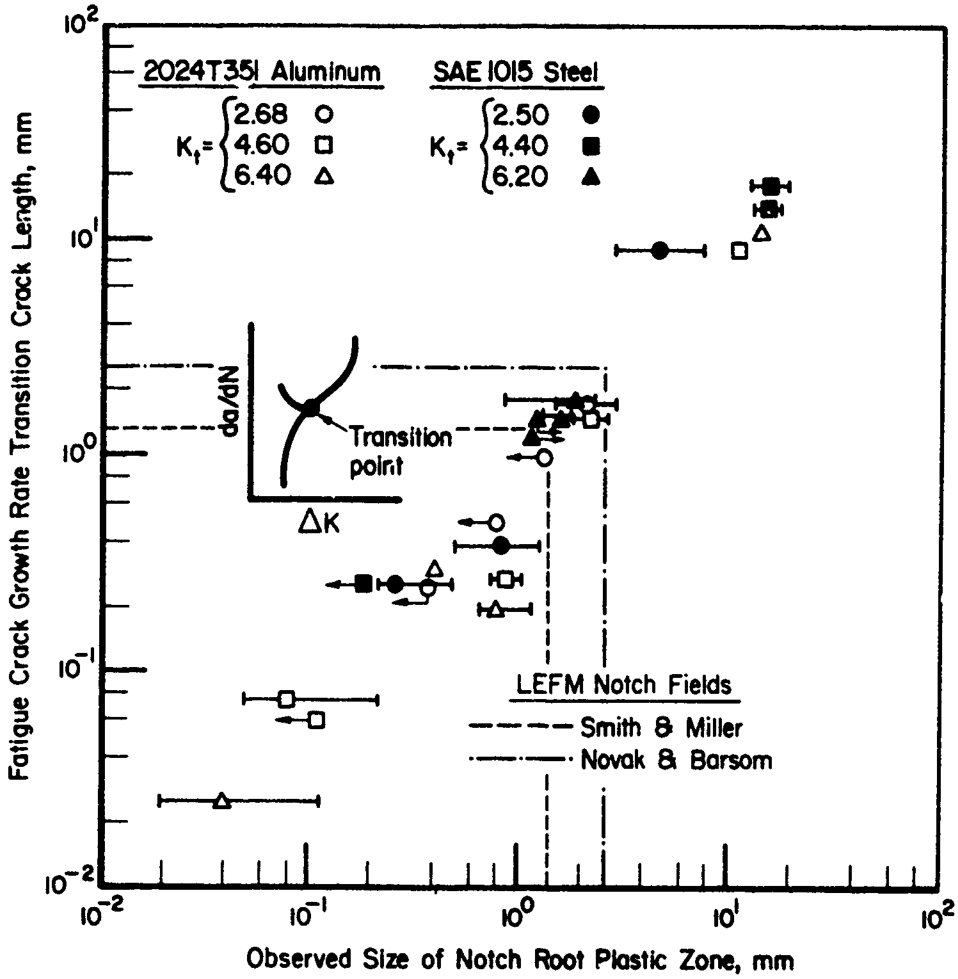

FIGURE 4.8. PREDICTED CRACK LENGTH FOR TRANSITION TO LONG CRACK BEHAVIOR BASED ON CONCEPT OF INELASTIC NOTCH FIELD DISPLACEMENT CONTROLLED GROWTH (SEE FIGURE 3.16 - A 1:1 CORRESPONDENCE IS AN EXACT PREDICTION) (Leis [190,205])

does not uniquely characterize the notch field. Clearly LEFM has obvious limitations when inelastic action occurs at notches. So also do models such as Equations (4.2) and (4.4) which rely on a load controlled crack whose plastic field is dominated over the plastic field of the notch.*

*Because Equation (4.2) includes an empirical calibration that may be a function of root radius and applied stress, this limitation exists only to the extent that the model as postulated did not address significant inelastic action.

5. MATHEMATICAL ANALYSES OF THE SHORT CRACK PROBLEM

This section addresses the various attempts that have been made to develop predictive models for the fatigue crack growth rates of short cracks. First, because the short crack problem per se is directly connected with the inability of relations based on conventional linear elastic fracture mechanics to provide adequate predictions, the basis of LEFM is discussed to illuminate the reasons why this might be expected. Next, the many analysis approaches that are based upon semi-empirical modifications to otherwise LEFM procedures will be discussed. Finally, those few attempts to model the key features of the short crack problem--residual crack tip plasticity and crack closure--will be discussed.

5.1 Some Fundamental Considerations in Fracture and Fatigue

The research that first decisively revealed they key role of the stress intensity factor in fatigue crack growth is generally credited to the 1963 publication of Paris and Erdogan [13]. They analyzed crack growth data from center cracked panels of a high strength aluminum alloy under two different types of loading conditions. One condition was a remote tension, the other a concentrated force acting on the crack surface. When the load is cycled between constant values, the first of these produces K values that increase with crack length, while the second gives K values that decrease with crack length. Because these data could be consolidated by ΔK, the use of this parameter as the driving force for fatigue cracking was well on its way to being established. It is significant that virtually all verifications of the applicability of LEFM in fatigue have been of a like empirical nature.

As reviewed, for example by Hertzberg [48], it is widely recognized that while fatigue crack growth rates could often be represented by a simple relation of the form

$$\frac{da}{dN} = C\ (\Delta K)^m \qquad (5.1)$$

effective data correlations over the complete range of ΔK values require a modification of this relation. At one extreme, the marked increase in growth at high ΔK values has led to the Forman relation

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K} \tag{5.2}$$

where K_c is the fracture toughness of the material. At the opposite extreme, Equation (5.1) is similarly violated due to threshold behavior. In this regime the relation

$$\frac{da}{dN} = C\left[\Delta K - (\Delta K)_{th}\right]^m \tag{5.3}$$

has been used where $(\Delta K)_{th}$ denotes the limiting or threshold value of the stress intensity factor. Relations combining the high and low ΔK effects also exist; for example, that of McEvily and Groeger [68]. However, even these are inadequate for coping with the influence of load interactions.

The nonlinear effect of load interactions is most clearly seen from the crack growth retardation that follows the imposition of a peak overload in a constant amplitude cyclic loading series. Two mechanical explanations have been suggested to explain this effect: residual compressive plastic stresses and crack closure. The semi-empirical model of Willenborg, et al [22] exemplifies those based upon the former mechanism while that of Elber [19], the latter. It is more likely that both effects occur simultaneously and, as discussed later in this section, some models have been offered that recognize this. However, it has not yet been possible to obtain closed form relations such as Equations (5.1), (5.2), and (5.3) above for conditions in which arbitrarily varying load sequences occur.

Further discussion on the predictive ability of LEFM-based fatigue crack growth models is beyond the scope of this report. It will suffice at this point to remark that, just as the foregoing suggests, such models do not perfectly mirror even long crack behavior. As has been pointed out emphatically in the earlier sections of this report, conventional fatigue crack growth models work because, being semi-empirical, experimental results can be predicted when similitude exists. This is true despite the fact that the basic assumptions of LEFM are violated for fatigue. To understand this, consider the modern view of linear elastic fracture mechanics that is contained in Figure 5.1 [162].

For a cracked body that everywhere obeys a linear elastic stress-strain law (see insert in Figure 5.1), the stresses in the near neighborhood of the crack tip can always be expressed in terms of a polar coordinate system (r, θ) with origin at the crack tip as

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} F_{ij}(\theta) + \ldots\ldots \tag{5.4}$$

where the omitted terms are of higher order in r. For small values of r (i.e., very near the crack tip), only the first term is significant. Then, the remote stresses, the crack length, and the external dimensions of the cracked body will affect the stresses at the crack tip only through the parameter K, the stress intensity factor. More definitely, there will be a region having the characteristic dimension D in which the first term of the series is a sufficiently good approximation to the actual stresses to any arbitrary degree of accuracy required. This region can be called the "K-dominant" region.

Referring again to Figure 5.1, note the presence of the inelastic region with a characteristic dimension R surrounding the crack. In a crack growth event, the inelastic processes would be associated with crack tip plasticity and, in a ductile material, with hole initiation, growth, and coalescence. In fatigue, the inelastic processes are the sliding off events stemming from the nature of fatigue crack growth on the microscale together with, of course, the inevitable crack tip plasticity. Regardless, because the assumption of linear elastic behavior in the region $r < R$ is invalid, it is not possible to directly characterize the fracture process using a linear elastic formulation. However, this is not necessary provided the inelastic region is contained in the K-dominant region. That is, if $R < D$, it can be argued that any event occurring within the inelastic region is controlled by the deformation in the surrounding K-dominant region. Consequently, if crack growth occurs, it must do so in a manner that is controlled by the stress intensity factor.

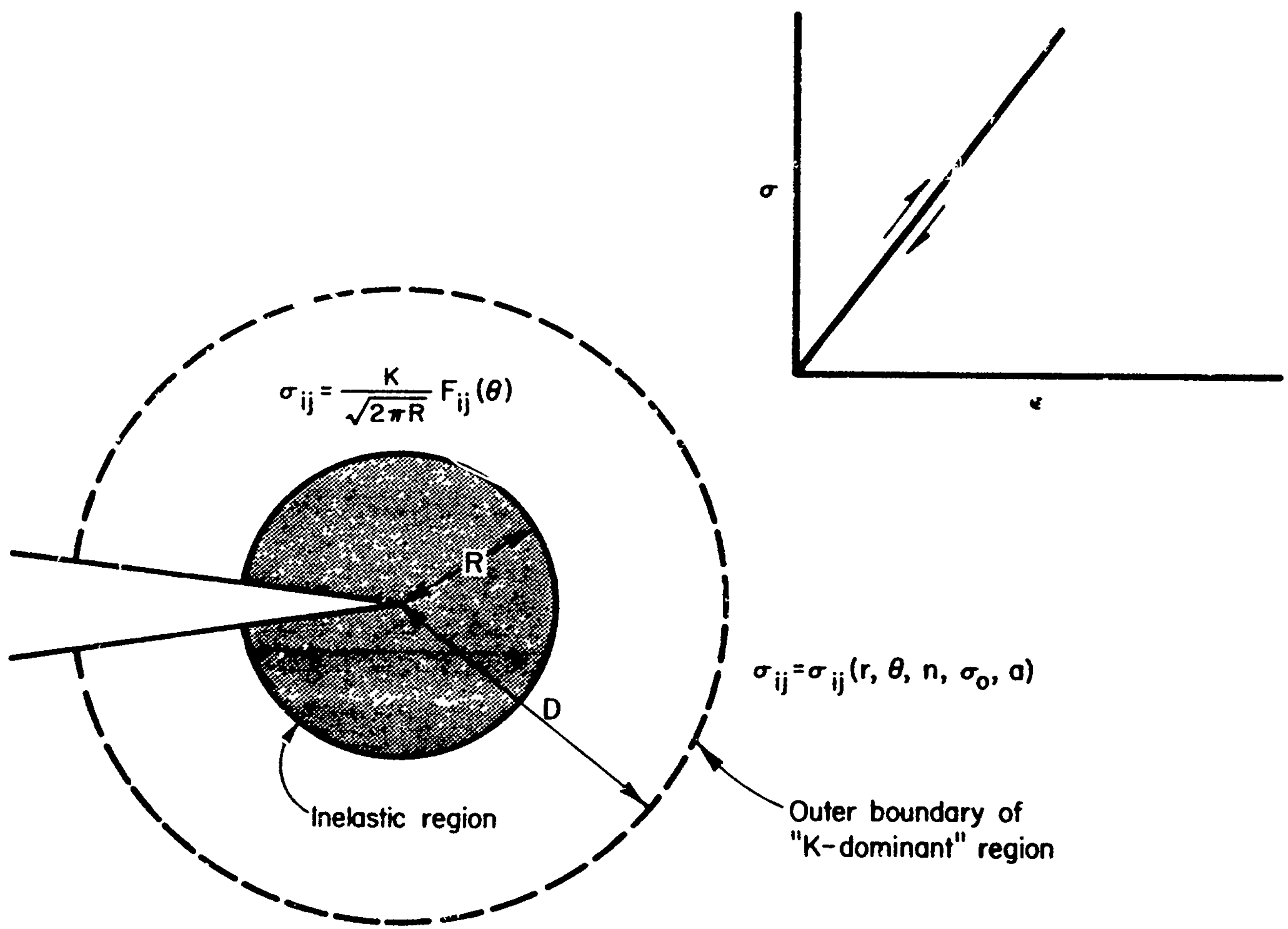

FIGURE 5.1. BASIS OF LINEAR ELASTIC FRACTURE MECHANICS (Kanninen, et al [162])

This argument has been used to neatly justify the use of basic LEFM relation $K = K_c$ for the onset of crack propagation. It could equally well be used to infer Equation (5.1). However, there is one objection. If the crack has propagated over some distance, it will have created a zone of residual plasticity over that distance. The condition that $R < D$ will then be inevitably lost whereupon the argument given above is not applicable.*

Again, Equation (5.1) is nonetheless unique because of the tacit invocation of similitude. For the short crack problem where similitude does not usually exist, it can readily be seen why the LEFM-based relations are defeated.

Because of the clear necessity to directly incorporate crack tip plasticity if more realistic results are to be achieved, several investigators have turned to the J-integral parameter. Hence, it is important to understand the basis of J in nonlinear fracture mechanics. As discussed by Kanninen, et al [162], an analogous argument to that given above for the K parameter can be followed. The argument is illustrated in Figure 5.2.

For the power law hardening material (see insert in Figure 5.2), the crack tip stresses can always be expressed as

$$\sigma_{ij} = J^{\frac{1}{n+1}} r^{-\frac{1}{n+1}} G_{ij}(\theta,n) + \ldots \tag{5.5}$$

where n is a property of the stress-strain curve called the strain hardening index and, as above, the omitted terms are of higher order in r. Thus, there is a "J-dominant" region which, if it contained the inelastic region at the crack tip, would allow J to control the events associated with crack growth.

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*The dimension D is a geometry-dependent value that is difficult to estimate precisely. Commonly, the requirement is stated in terms of R being small in comparison to the crack length and other dimensions of the body. A simple estimate can then be based upon choosing R as the point at which the normal stress on the prolongation of the crack length reaches the yield stress, σ_y. Then, Δa should not exceed $(k_{max}/\sigma_y)^2/2\pi$ for the LEFM condition to apply.

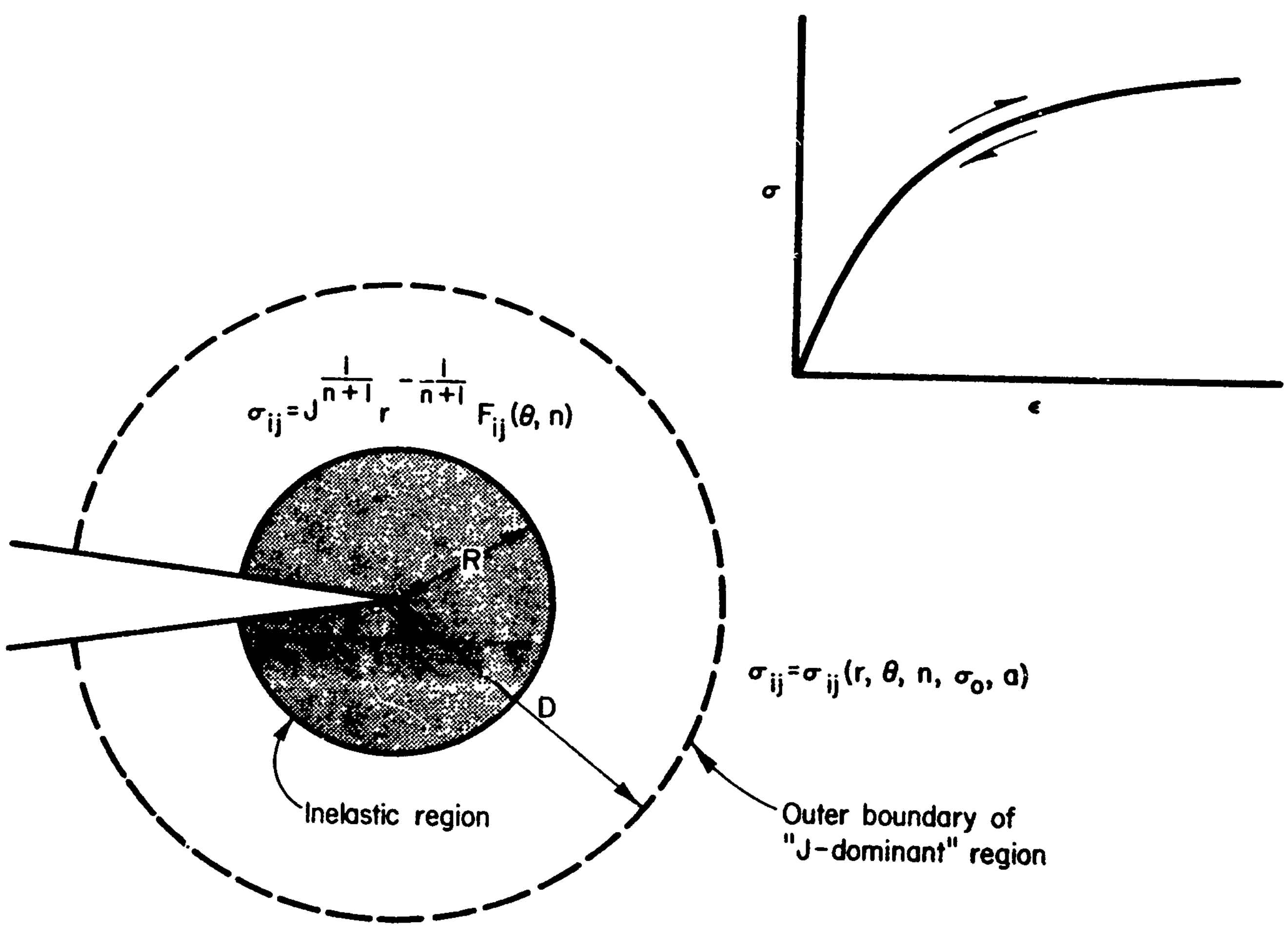


FIGURE 5.2. BASIS FOR THE USE OF THE J-INTEGRAL IN PLASTIC FRACTURE MECHANICS (Kanninen, et al [162])

This leads to the use of $J = J_R(\Delta a)$ in elastic-plastic fracture mechanics; i.e., a "resistance curve" approach in which J_R is taken as a material property. Because the inelastic region is <u>not</u> the plastic region here, this result would seemingly allow a similar relation to be used for fatigue. However, as the following indicates, this is not true.

There is a requirement for deformation plasticity (i.e., reversible loading) to be satisfied for Equation (5.5) to be valid. Because real material deformation is not generally reversible, continued crack growth inexorably invalidates the use of J. This becomes apparent in the elastic-plastic fracture mechanics approach known as the tearing instability analysis where there is a limit on the extent of crack growth for which the theory is valid [162]. Consequently, while the use of nonlinear fracture mechanics parameters in fatigue may ameliorate the problem somewhat, the basic limitation arising in LEFM is not overcome. Obviously, this will also be true in the short crack domain as well.

Before turning to specific approaches to the short crack problem, one possibly useful finding of elastic-plastic fracture mechanics might be noted. This is the observation that stable crack propagation in ductile materials, after some initial transient, occurs with a virtually constant crack opening profile. This was apparently first suggested by deKoning [69] in work on aluminum and subsequently reinforced for stainless steel in work at Battelle [137] and elsewhere [196] . It follows from these observations that the instantaneous crack tip opening displacement (CTOD) must be very nearly a constant. While this fact has not been directly related to subcritical crack growth, it does lend credence to the possibility that the CTOD can be an effective measure of the crack driving force for fatigue. Indeed, since under LEFM conditions K and CTOD are directly related, no loss of generality arises from the use of CTOD. At the same time the intriguing possibility exists that it may offer the basis for a more broadly applicable elastic-plastic fatigue relation.

To summarize, the foregoing introductory discussion has been intended to show that the short crack problem is not a unique departure from the LEFM-based fatigue crack growth relations. To the contrary, there are a great many instances in which modifications to these relations are needed to give acceptable conclusions. The reason is simply that the basic LEFM assumptions are inherently violated for a growing crack. It follows that, when advantage cannot be taken of similitude, more intricate analysis approaches are needed. Specifically, attention must be given to residual crack tip plasticity and to its manifestations; e.g., crack closure. The following subsections will address current progress in so doing.

5.2 Semi-Empirical Analysis Models for Short Cracks

Analysis approaches addressed to short cracks can be divided into two main categories. One includes those attempts that have incorporated some modification or correction into a conventional fracture mechanics formulation. These generally semi-empirical treatments are described in this subsection. Because these have already been described in connection with the phenomenological work reviewed in the preceding sections of this report, only a brief description is given here for completeness. The second category, described in more detail, includes the more rigorous approaches that attempt to treat short crack fatigue from the first principles of continuum mechanics. More particularly, because the essence of the problem is the presence of plastic deformation, these lie within the domain of elastic-plastic fracture mechanics. The rigorous implementation of elastic-plastic analysis therefore distinguishes them from those discussed next.

El Haddad, et al [111] have recognized that a generalization of conventional fatigue crack growth rate predictive techniques is necessary and have chosen to provide one within the confines of deformation plasticity. Specifically, they have suggested the addition of a small pseudo crack length, ℓ_0, to the actual crack length. Because the material constant ℓ_0 is small, long crack behavior is unchanged, as is essential in any such approach. However, for small crack lengths, the use of ℓ_0 increases K. This escalates the crack growth rate, as required to match the short crack effect.

The approach of El Haddad, et al can be criticized on both fundamental and pragmatic grounds. They argue that ℓ_0 can be related to the threshold stress intensity and the fatigue limit. If this interpretation is correct, the physical meaning of ℓ_0 must be that of an inherent defect length in the material which dominates the specimen response when the artificial crack length is of the order of ℓ_0. But, it surely is disingenuous to suppose that such defects are both non-interacting and always located so as to increase the artificial crack length. While it would be permissible to limit the smallest crack length to be ℓ_0, it cannot logically be incorporated as an additive term.

On a practical basis, the approach of El Haddad, et al similarly appears to be wanting. For example, their approach fails to consolidate the short crack data for mild steel developed by Leis and Forte [179]. This version of the El Haddad, et al approach is based upon the use of the J-integral, as originally suggested by Dowling [46], in an effort to formulate a plastic fracture mechanics formalism. But, the pseudo crack length concept, even when enhanced by the use of J, is not in correspondence with these data. Similar discouraging results for other J-based approaches have been obtained by Leis [190]. On a theoretical basis, the argument given in the above for the use of J in elastic-plastic fracture mechanics clearly shows why the substitution of J for K as the crack driving force parameter in fatigue is doomed in any event.

Another effort that institutes J as the controlling parameter is that of Sadananda and Shahinian [138]. While they recognize that J is not theoretically valid (see the discussion on J in section 5.1 above), they conclude that J is superior to K, particularly at high temperatures where nonlinear effects are more pronounced. But, in their work, $(\Delta K)^n$ is simply replaced by $(\Delta J)^{n/2}$ whereupon it can be concluded that it is once again similitude that is the redeeming feature.

As has been stated at many places in this report, fatigue analyses based on the stress intensity factor violate the basic assumptions for the use of this parameter. This usage is only redeemed by satisfying the principle of similitude. The primary reason is that LEFM neglects a key factor controlling fatigue crack growth: the plastic deformation that surrounds the crack tip. In load interaction situations it is the residual plastic deformation that has accrued from previous load cycles that is important. For short cracks, it is the plastic deformation arising from a stress riser that engulfs the crack that is the critical factor missing from LEFM-based treatments. Regardless, analyses specifically admitting elastic-plastic deformation are called for.

5.3 Elastic-Plastic Fracture Mechanics Analyses for Short Cracks

The various possible representations of crack tip plasticity that are available for use in fatigue analyses are listed in Table 5.1. These are ordered in ascending order of accuracy and, inevitably, in descending order of convenience. The semi-empirical extensions of LEFM, the first listed entry, have already been discussed in the above. In this section, the two varients of the strip yield zone representation of crack tip plasticity will be discussed along with the very refined--but quite cumbersome--analyses employing elastic-plastic finite element models.

In contrast to the previous paper categorizations reviewed in this report, relatively few elastic-plastic mathematical model developments have been attempted for the short crack problem. In terms of Table 5.1, those worthy of note are the approaches using an extended version of the Dugdale (collinear) strip yield model and those using elastic-plastic finite element models. In the former group are Kanninen and coworkers at Battelle [164,193], Newman [202], and Seeger and coworkers [29,71,123], while in the latter is the work of Trantina, et al [194,195].* It might be noted that while the use of an inclined strip yield model has yet to be applied to the short crack problem per se, as the work of Kanninen and Atkinson [139] shows, this approach offers an intriguing compromise between a realistic representation of plane stress plastic deformation and computational convenience that warrant serious consideration in future attempts.

5.3.1 The Work of Battelle

Recent work on stable crack growth in ductile materials performed at Battelle [137,162] has revealed that the instantaneous crack tip opening displacement (CTOD) plays a key role in this process. Specifically, after some

*Newman's ligament model [202] is based upon his earlier finite element analyses of fatigue crack growth; e.g., reference (44). However, he has apparently not applied the finite element method directly to the short crack problem as Trantina, et al have.

TABLE 5.1. ANALYTICAL PROCEDURES FOR THE PREDICTIONS OF FATIGUE CRACK GROWTH UNDER SPECTRUM LOADING

Technique	Strong Points	Weak Points
Semiempirical extensions of linear elastic fracture mechanics based upon K or J	Gives simple relations that are easy to apply; offers insight into controlling mechanisms.	Lack of firm fundamental basis; difficulty in treating complicated histories; cannot easily be generalized; short crack effect not predicted.
Dugdale strip yield model with critical crack-opening displacement criterion	Gives closed-form result for steady-state growth rate; can be extended to include load history effects.	Can distinguish load history effects and crack closure only by use of unrepresentative crack tip plasticity
Inclinded strip-yield superdislocation model with critical crack-opening displacement criterion	Gives realistic plastic zone representation; plastic deformation in different load cycles is distinguishable; closure effects handled directly.	Predictions for complicated load histories may require lengthy computations.
Elastic-plastic finite-element analysis	Highly accurate; can be used to treat wide variety of situations useful for examining details of crack growth process.	Very time-consuming computations required; crack extension criterion not now established; crack advance increment is arbitrary.

initial transient, the CTOD is observed to take on a constant value which is maintained over extended amounts of crack growth. Clearly these observations were not made under identical conditions to those occurring in fatigue. Nevertheless, it could be that enough similarity exists that subcritical crack growth could also be governed by the CTOD parameters. Further, such a point of view is entirely compatible with the crack closure effect. The idea of developing a fatigue crack growth model based upon CTOD is certainly not new. The difference in the approach being pursued at Battelle is that it inherently includes a nonlinear relation between CTOD and the crack advance increment in contrast to the linear relations invariably used in prior attempts.

To explore the use of CTOD as a governing parameter, the Battelle approach [164,193] considered that a valid long crack fatigue crack growth relation is of the form given by Equation (5.1). It might be noted that any other valid form containing K or J would serve just as well for the purpose of the development that follows. This is because under small scale yielding conditions both K and J are related to CTOD. These relations are

$$\delta = \alpha \frac{K^2}{E\sigma_y} \tag{5.6}$$

$$= d_n \frac{J}{\sigma_y} \tag{5.7}$$

where,

δ = CTOD

E = Elastic modulus

σ_y = Yield strength

α = a numerical constant of order unity

d_n = a function of strain hardening exponent and stress state.

Equation (5.7) will not be used in the following but is shown to suggest that the procedure could equally well be based upon J.

Equations (5.1) and (5.6) can be combined to get

$$\frac{da}{dN} = C' \left[\delta_{max}^{1/2} - \delta_{min}^{1/2}\right]^m \tag{5.8}$$

where $C' = C(\frac{E\sigma_y}{\alpha})^{m/2}$. Now, it can be hypothesized that Equation (5.8), derived on the basis of the LEFM relationship of Equation (5.6), is valid beyond the small scale yielding regime. This assumption has a basis in other work at Battelle where it was shown (although under a different cracking mechanism) that even when accepted elastic-plastic fracture mechanics parameters--e.g., the J-integral--become invalid, CTOD remained an acceptable crack growth parameter.

The inevitable question then is how does one use a crack growth model such as Equation (5.8) since CTOD cannot be really measured in the laboratory with sufficient precision. This argument has persistently been used against all CTOD based models. The irony in such an argument lies in the fact that K, the commonly used parameter in FCG studies, cannot be measured either. It can only be inferred using experimental measurements via some analysis. CTOD can be found in a similar manner except that the analysis in this case is more involved and generally applicable handbook solutions are not as readily available.

A simple way of obtaining an estimate of CTOD is via a Dugdale model which is a special case of the more general pseudoplastic modelling approach discussed by Atkinson and Kanninen [139]. For geometries other than simple center-cracked panels or edge-cracked panels, Dugdale solutions are not readily available in the literature. However, any linear elastic solution procedure, numerical or exact, which can be applied to a given geometry to obtain stress intensity factors witn constant traction applied over any given portion of the crack surface can also be used to perform the Dugdale analysis. This, of course, is because the Dugdale model is obtained by simple superposition of such linear elastic solutions. But, for the specimen geometry chosen for this work, compliance functions necessary for performing a weight function analysis were not available. Therefore, all analyses were performed using a finite element method employing planar eight noded isoparametric elements in conjunction with quarter point elements to model the crack tip stress singularity.

The superposition principle used in the Dugdale analysis is illustrated in Figure 5.3. The analysis first involves determination of K for Problem 1. Then, a series of solutions for Problem 2 is obtained with varying

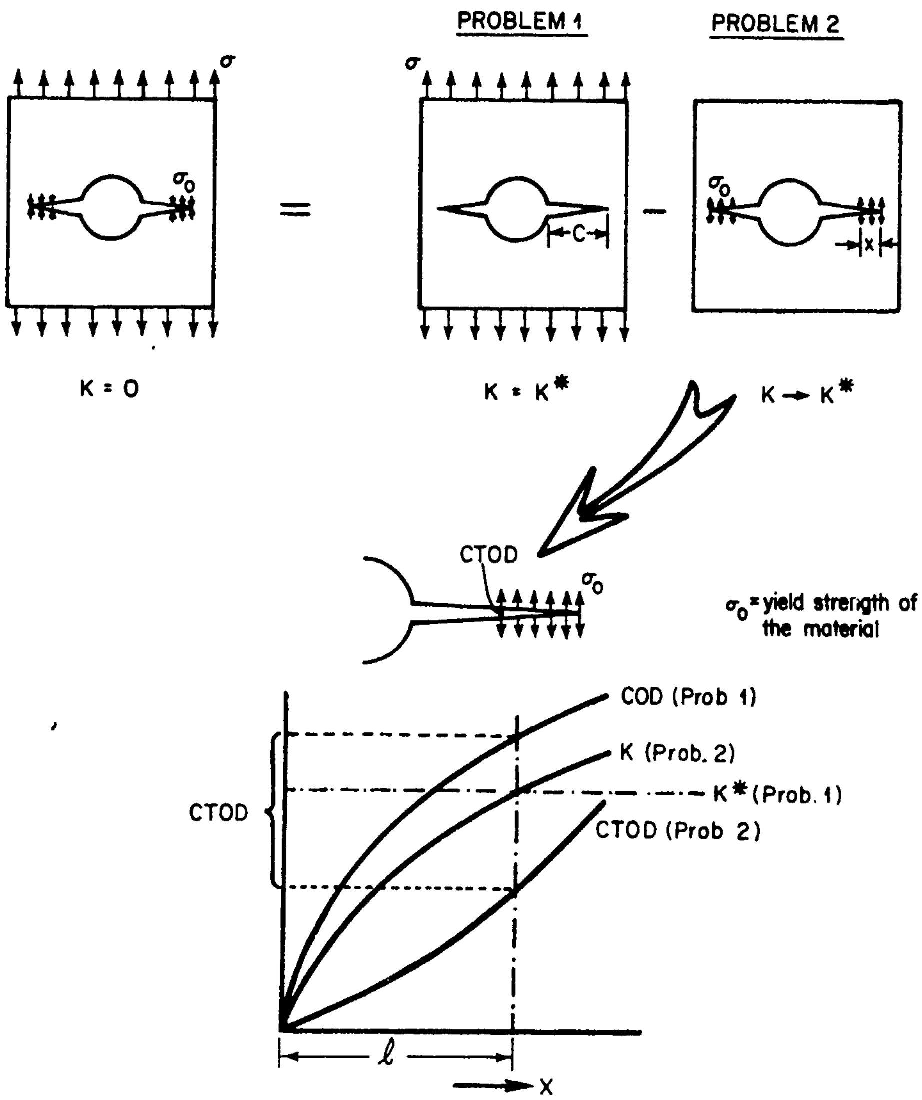


FIGURE 5.3. FCG RATE PREDICTIONS USING CTOD AND COMPARISON WITH EXPERIMENT AS WELL AS LEFM PREDICTIONS

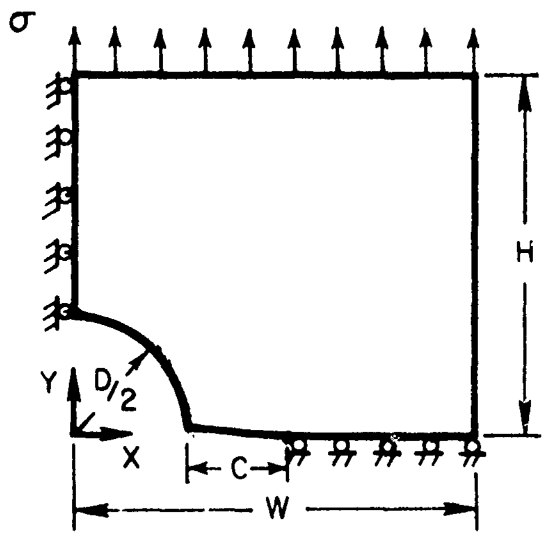
distance x over which traction equal to the yield strength of the material is applied. When the length x becomes such that the K value of Problem 2 is equal to the K of Problem 1, superposition of the two problems provides the zero K required for the Dugdale model. The "singularity cancelling" value of x is the Dugdale plastic zone ℓ for a physical crack length $a = (c - \ell)$. The CTOD is found by superimposing the vertical displacements of the two problems at $x - \ell$ as shown in Figure 5.3.

The specimen geometry used in the experiments analyzed using this procedure is symmetrical with respect to both the horizontal and the vertical axes passing through the center of the hole. If the cracks also grew symmetrically on both sides of the hole, the analysis could be performed using only one quarter of the domain (as shown in Figure 5.4) by imposing appropriate symmetry boundary conditions. This analysis was performed and is referred to as case (A) in the numerical results which follow (Figure 5.5). However, in the actual experiment one of the cracks initiated and grew to approximately 5.08 mm (0.2 in) before the second crack appeared. Therefore, until the two cracks became of equal length, the geometry was symmetrical only about the horizontal axis. Consequently, analyses using this geometry (Case B) were performed using one-half of the domain as shown in Figure 5.4.

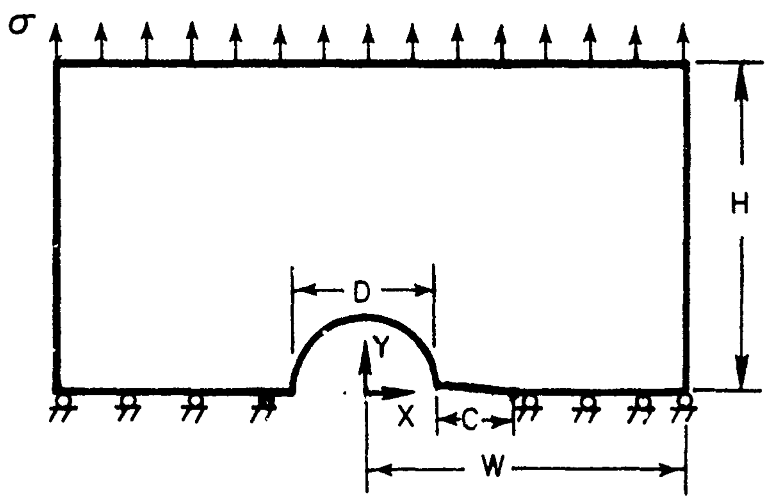
Crack growth rate predictions using Equation (5.8) can be made in a straightforward manner. All that is needed are the values of constants C and m. These were taken to be

$$C = 9.1 \times 10^{-9}$$
$$m = 2.62$$

The FCG rate predictions made using the results of analyses of Cases A and B, along with the experimental results, are shown in Figure 5.5. It can clearly be seen from these results that the CTOD-based analysis procedure developed here gives a significantly better prediction of the experimental results than does the LEFM-based procedure. Of more significance, the nonconservative nature of the conventional approach is removed.



(a) One Quarter of Domain for Case A



(b) One Half of Domain for Case B

FIGURE 5.4. THE GEOMETRICAL CONFIGURATIONS FOR ANALYSIS (σ = 27.08 KSI, W = 1.5 IN., H = 1.25 IN., D = 0.5 IN.)

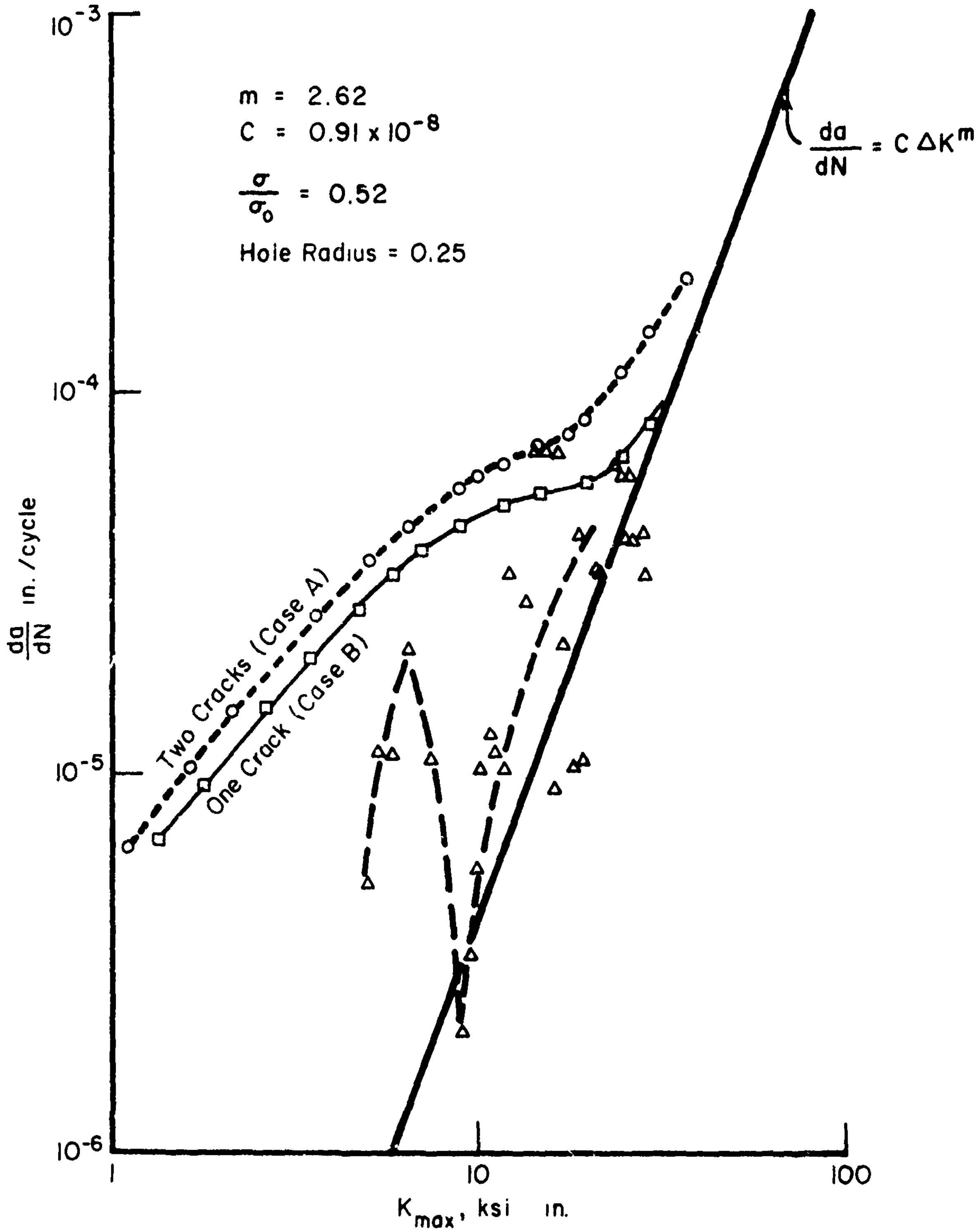


FIGURE 5.5 CRACK GROWTH RATE PREDICTIONS AND EXPERIMENTAL RESULT

5.3.2 The Work of Newman

The work just described was based upon the use of the Dugdale crack tip plasticity model in a simple manner to investigate the effectiveness of using a CTOD criterion for the growth of short fatigue cracks. In so doing it was recognized that some key effects--e.g., crack closure--were omitted. That this is not precluded through the adoption of a Dugdale model is clearly shown in the important work performed by Newman [44,156] who has devised a "ligament model" generalization of the Dugdale model, via a finite element calibration. This work has recently been focussed on the short crack problem [202], as discussed in the following.

Newman has developed a model by modifying the Dugdale model to leave plastically deformed material along the crack faces as the crack grows. This is shown conceptually in Figure 5.6 [202]. His purpose was to study the related effects of small crack growth rates and of large cracks under the load reduction schemes used to determine threshold stress intensity values. As usual in the NASA orientation in Newman's work, emphasis was placed upon crack closure.

Newman cites as the primary advantage of the Dugdale model that linear superposition is valid even when such ostensibly nonlinear effects as crack closure are included. This is true because the crack closure effects take place only from residual plasticity in the line of the crack. By leaving plastically deformed material behind the crack in this way, the crack surface displacements used to calculate contact (closure) stresses under cyclic loading are influenced by the plastic yielding both ahead and behind the crack tip. Specifically, bar elements, assumed to behave like rigid perfectly-plastic materials, are used (see Figure 5.6). At any applied stress level, these elements are intact ahead of the crack tip or are broken to represent the residual plasticity behind the crack tip.

It is important to recognize that the broken elements carry compressive loads only, and then only if they are in contact. Those elements that are not in contact apparently do not affect the calculation in any way. They are used simply to calculate crack-opening stresses (both crack tip

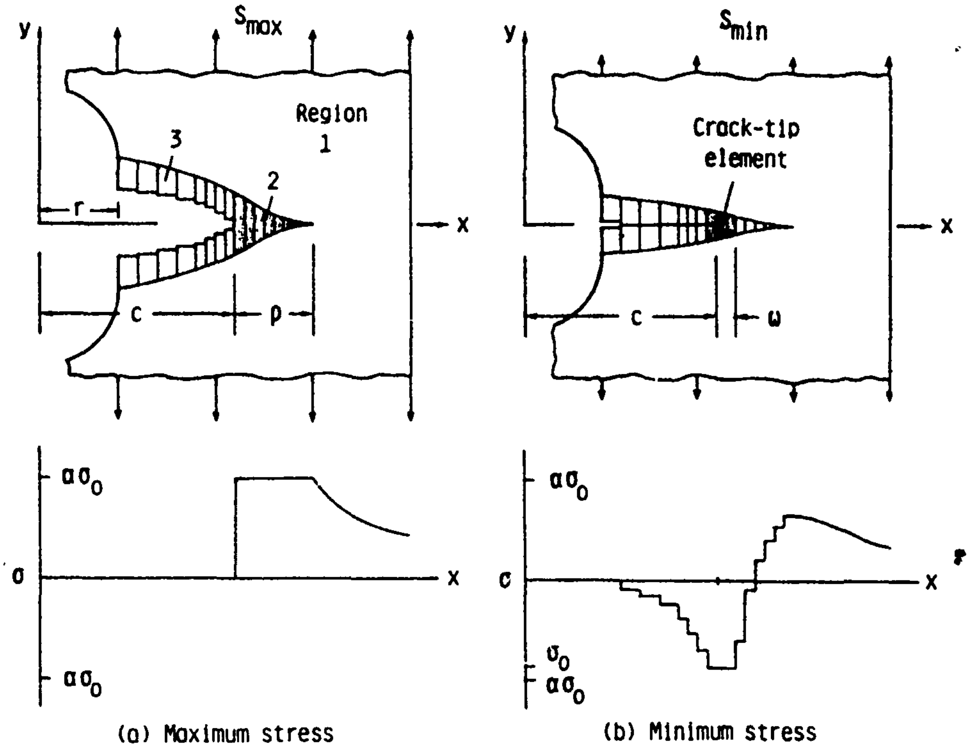


FIGURE 5.6. CRACK-SURFACE DISPLACEMENTS AND STRESS DISTRIBUTIONS ALONG CRACK LINE (Newman [202])

closure and closure elsewhere on the crack edges can be handled) for use in determining an "effective" ΔK value in the manner suggested by Elber; i.e., to determine ΔK_{eff} where

$$\Delta K_{eff} = \left[\frac{\sigma_{max} - \sigma_o}{\sigma_{max} - \sigma_{min}}\right] \Delta K \tag{5.9}$$

where σ_o denotes the crack opening stress. Note that, although not specifically indicated here, Newman uses a plasticity corrected K value in his calculations.

Results obtained for short crack growth by Newman's analysis are shown in Figures 5.7 and 5.8 for steel specimens and in Figure 5.9 for an aluminum specimen. Calculations were made for a center-cracked tension panel and for cracks emanating from a hole in the former results while the latter shows only the result for the specimen with a hole. From these very encouraging results Newman concludes that the short crack effect is a result of the differences in the crack closure effect between long and short cracks. In particular, at equal K values, the applied stress needed to open a small crack is less than that required to open a large crack. Consequently, the effective stress range is greater for small cracks. This, in turn, gives rise to the higher crack growth rates that exemplify the short crack effect.

5.3.3 The Approach of Seeger and Coworkers

The work of Seeger [29] and of Seeger and his coworkers, Fuhring and Hanel [33,71,123], is of great relevance to the small crack problem. The work consists of scholarly studies of the Dugdale model and its application to cyclic crack growth. Although Seeger et al have sought application primarily to variable amplitude loading and retardation, their work is equally applicable to the small crack problem.

By using a trapesoidal necking configuration suggested by Hahn and Rosenfield [15], Seeger established a solution procedure for the Dugdale model using a Ramberg-Osgood stress-strain equation. The solution is obtained by

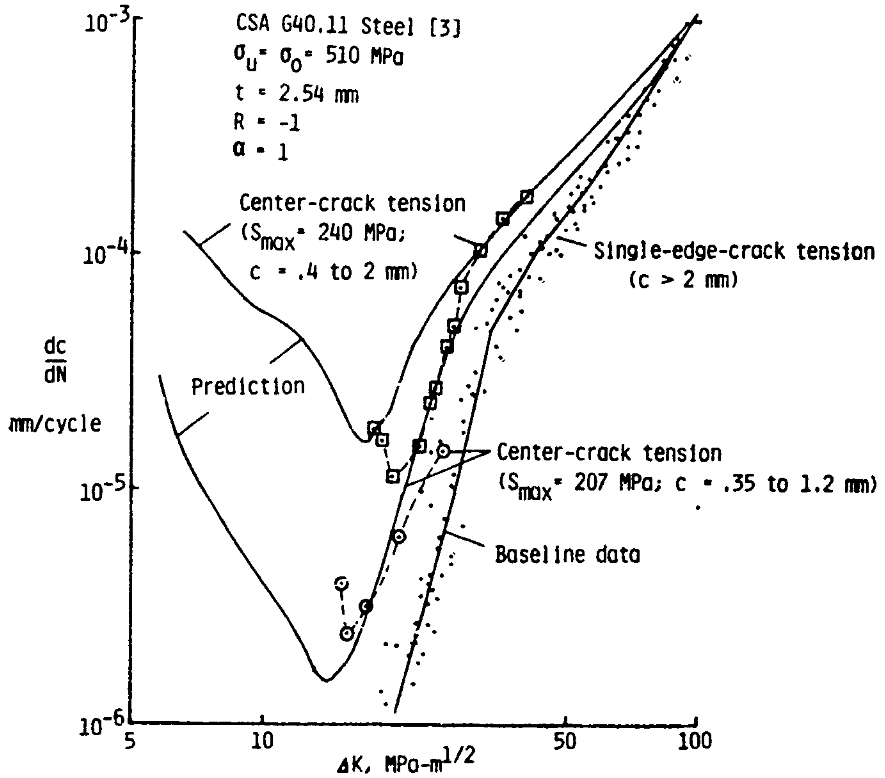


FIGURE 5.7. COMPARISON OF EXPERIMENTAL AND PREDICTED CRACK-GROWTH RATES FOR SMALL CRACKS IN CENTER-CRACK TENSION SPECIMENS SUBJECTED TO HIGH STRESS LEVELS (Newman [202])

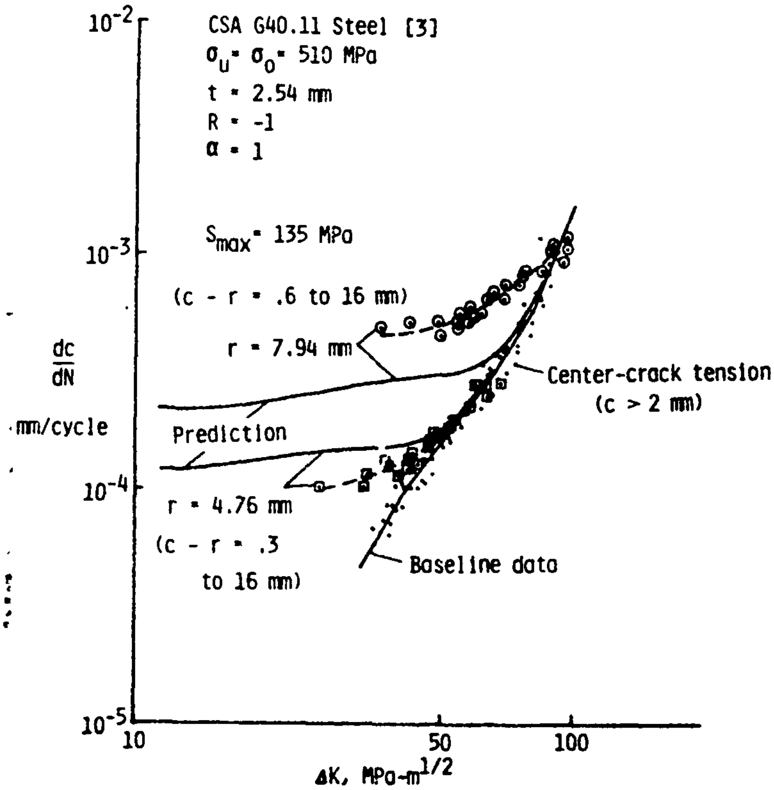


FIGURE 5.8. COMPARISON OF EXPERIMENTAL AND PREDICTED CRACK-GROWTH RATES FOR SMALL CRACKS EMANATING FROM A CIRCULAR HOLE IN STEEL SPECIMENS (Newman [202])

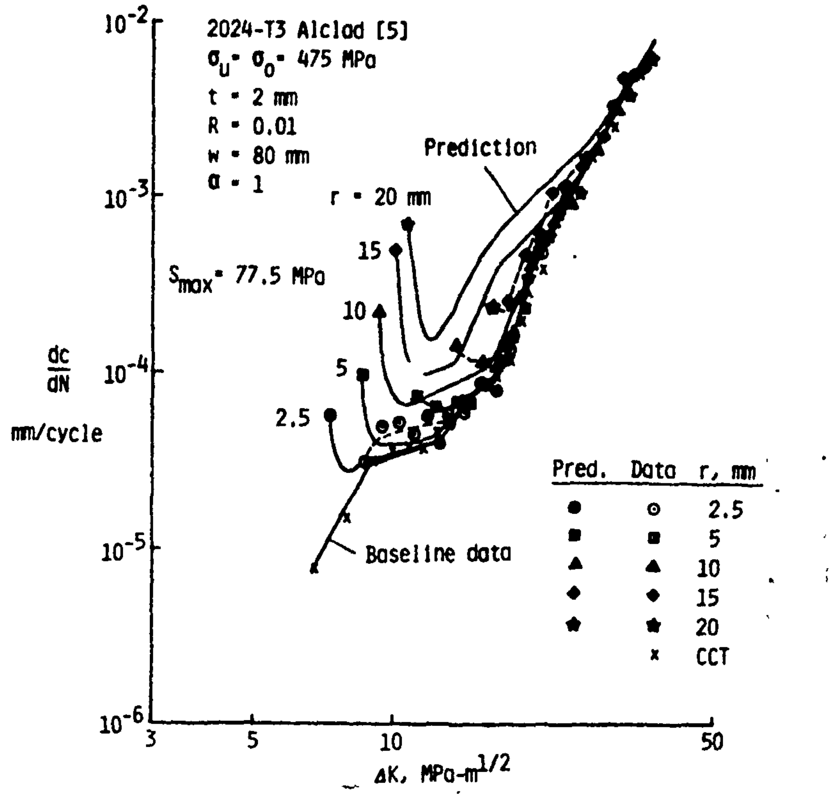


FIGURE 5.9. COMPARISON OF EXPERIMENTAL AND PREDICTED CRACK-GROWTH RATES FOR SMALL CRACKS EMANATING FROM A CIRCULAR HOLE IN ALUMINUM SPECIMENS (Newman [202])

iteration. Although this solution can be used for cyclic loading with closure (and in some cases has been used), in most of the work the material is treated as ideally plastic.

Fuhring shows that application of the Manson-Coffin fatigue equation essentially leads to a crack growth equation of the type

$$\frac{da}{dN} = C(\Delta CTOD)^m \tag{5.10}$$

For cases where ΔCTOD is uniquely related to the applied ΔK, this leads immediately to Equation (5.1). When ΔCTOD is not uniquely related to applied ΔK, the log da/dN - log ΔK plot will naturally be non-linear.

Use of Equation (5.10) requires calculation of ΔCTOD, which depends upon σ_{max}, σ_{min}, and closure effects that are brought into the Dugdale model in a natural way. A Dugdale solution for CTOD requires that elastic solutions in functional form be available for the crack under applied external loading and for the crack with forces on the crack edges. The only case (for finite size) for which these solutions exist is the center cracked plate of width W, crack size 2a, provided the solution for collinear cracks of size 2a and spacing W is accepted to be applicable. (The latter is accepted so widely that it needs no justification.)

Consider a crack grown naturally by fatigue. The halves of all previous plastic zones are still attached to the crack edges (Figure 5.10). Assume first that these previous plastic zones have never made contact. This means that the plastic zone at location X is equal to half the CTOD that existed when the crack tip was at location X.

When accounting for the Baushinger effect, ΔCTOD(X) can be calculated. Then, upon unloading

$$COD_{min}\ (X) = COD_{max}\ (X) - COD(X) - CTOD_a = X \tag{5.11}$$

If $COD_{min}(X) \leq 0$ the faces make contact and the Dugdale model prescribes that at such places a stress equal to $2\ \sigma_{ys}$ is transmitted. Since the material is ideally plastic the previous plastic zone $CTOD_a = X$ will be

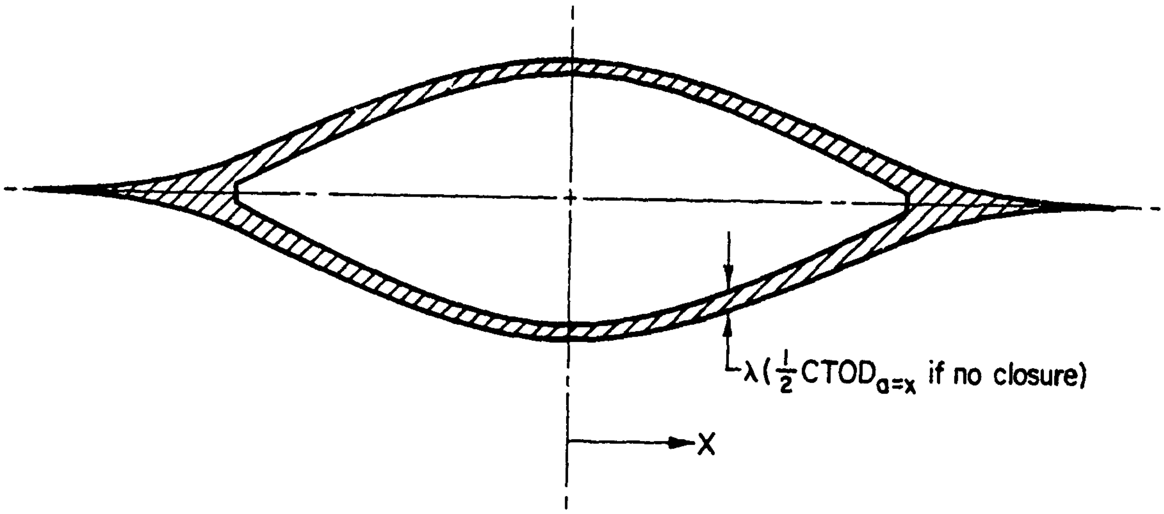


FIGURE 5.10. PRINCIPLE OF MODEL BY SEEGER, FUHRING, AND HANEL

squeezed at places of contact until $COD_{min}(X) = 0$. Thus the size of the plastic lip is reduced to

$$\lambda(X) = COD_{max}(X) - \Delta COD(X) \tag{5.12}$$

If contact occurs again during subsequent unloading cycles, a stress σ_{ys} will be transmitted even when $CTOD_{min}(X) = 0$. When $CTOD_{min}(X)$ would be less than zero (due to further unloading) $\lambda(X)$ would be squeezed further. By applying σ_{ys} over the area of contact and 2 σ_{ys} (Baushinger) over the reverse plastic zone, a Dugdale solution can be obtained for $CTOD_{min}$. Then $\Delta CTOD$ follows from

$$\Delta CTOD = CTOD_{max} - CTOD_{min} \tag{5.13}$$

In their numerical calculations for cycle by cycle loading, Seeger et al account for the so-called "memory effect" in much the same way as is done in fatigue analysis. This can be appreciated from Figure 5.11, where memory comes into action between 1 and 3. Figure 5.11 also shows that $\Delta CTOD$ is much reduced due to closure. As already mentioned, Seeger et al have applied this specifically to variable amplitude loading. Figures 5.12 and 5.13 show $\Delta CTOD$ during and following an overload.

Fuhring [71] proposed to obtain solutions for other configurations by using approximate solutions for plastic zone size and CTOD as follows. For the infinite plate under uniform tension we have:

$$\frac{CTOD}{a} = \frac{8\sigma_{ys}}{\pi E} \ln \sec \frac{\pi \beta \sigma}{2\sigma_{ys}} \tag{5.14}$$

and

$$\frac{r_p}{a} = \sec \frac{\pi \sigma}{2\sigma_{ys}} - 1 \tag{5.15}$$

Using series developments for small arguments, these equations provide

$$CTOD = \frac{K^2}{E\,\sigma_{ys}} \text{ and } r_p = \frac{\pi}{8} \frac{K^2}{\sigma_{ys}^2} \tag{5.16}$$

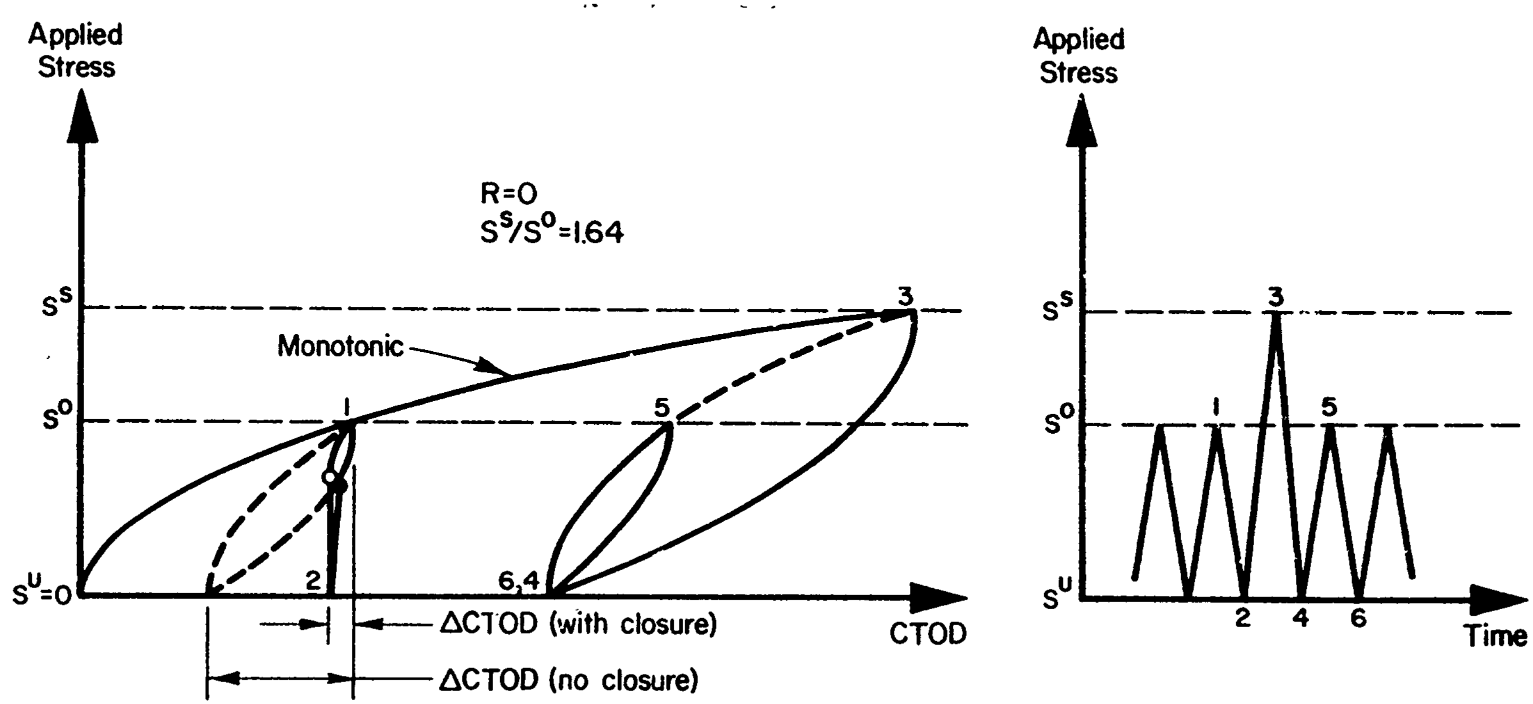


FIGURE 5.11. ΔCTOD BEFORE AND AFTER OVERLOAD (Fuhring [71])

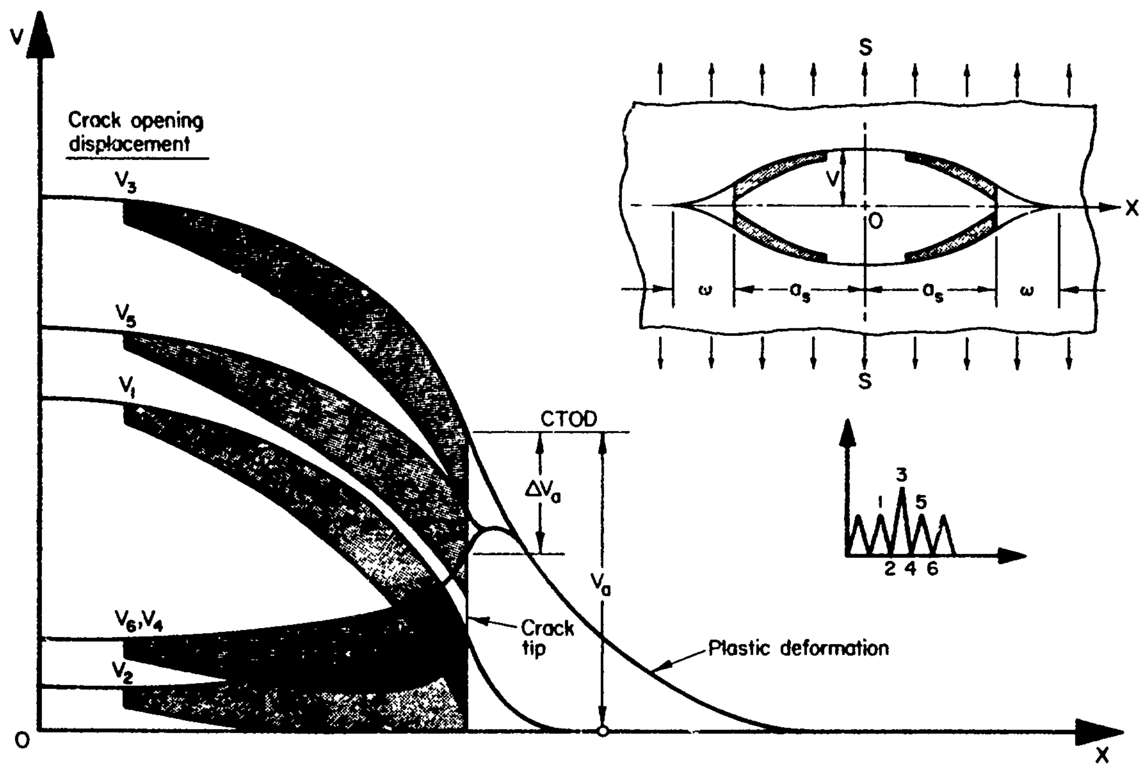


FIGURE 5.12. RESULTS OF FUHRING, SEEGER, AND HANEL MODEL

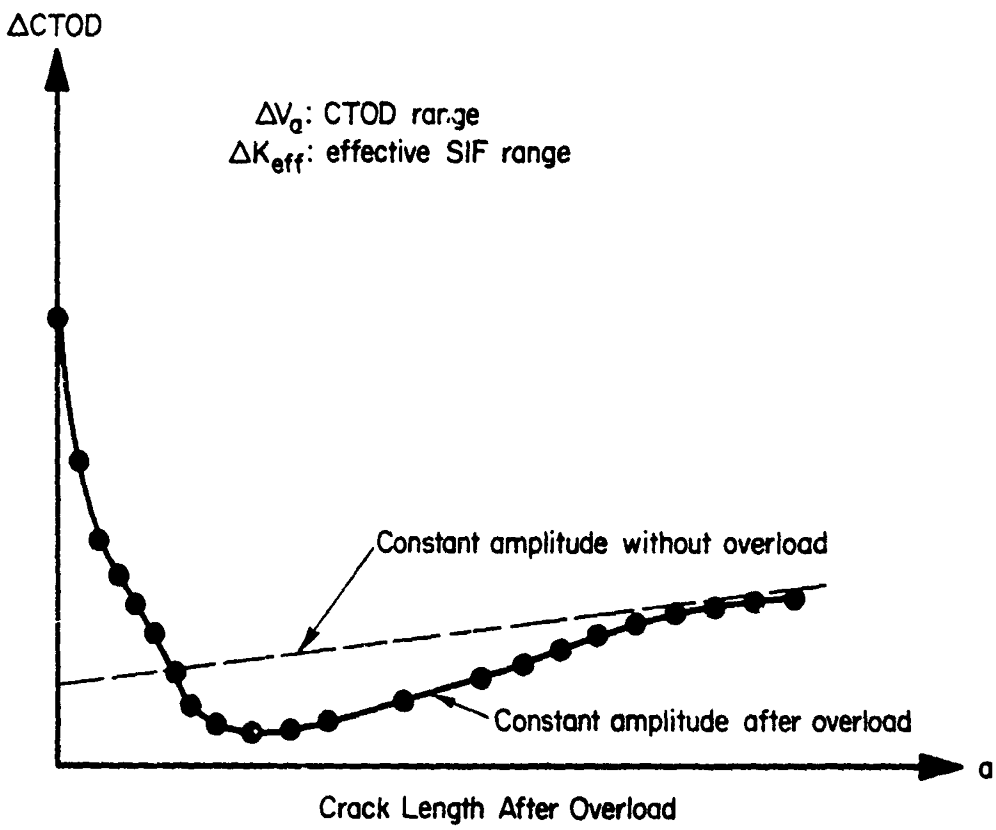


FIGURE 5.13. CHANGE OF ΔCTOD DURING CONSTANT AMPLITUDE LOADING WITH AND WITHOUT PRIOR OVERLOAD

Generalization of Equation (5.16) with $K = \beta\, \sigma\sqrt{\pi\, a}$ permits the equations to be rewritten as Equations (5.17) and (5.18) if β is close to 1:

$$\frac{CTOD}{a} = \frac{8\sigma_{ys}}{E} \ln \sec \frac{\pi\beta\sigma}{2\sigma_{ys}} \tag{5.17}$$

and

$$\frac{r_p}{a} = \sec \frac{\pi\sigma}{2\sigma_{ys}} - 1 \tag{5.18}$$

Fuhring also shows that this result follows automatically if the stress intensity due to crack edge loading has the form:

$$K_p = \frac{2p}{\pi a} \int_{X_1}^{X_2} \frac{C\, d\chi e}{\sqrt{1 - (\chi/a)^2}} \tag{5.19}$$

where p is the crack edge stress.

In order to assess the accuracy of Equations (5.17) and (5.18) Fuhring applied them to the center cracked panel and compared the results with those obtained using the exact solution for the colinear cracks. As shown in Figure 5.14, errors of less than 5 percent are obtained for a wide range of the relevant parameters.

It is not clear how Fuhring expects to obtain an expression for CTOD in such an approximate manner for the evaluation of Equations (5.11), (5.12), and (5.13). However, if closure is limited to an area close to the crack tip, the displacements are expressable through K and therefore the use of $\nu = \frac{K}{E}\sqrt{\pi a}$ would be justifiable.

Clearly, the "short" crack problem is associated with the plastic deformation not associated with the crack per se, but with the consequent residual stresses and closure effects. Therefore, no model or analysis can explain the short crack behavior unless closure and residual stress effects are accounted for. The significance of the work of Seeger et al is in the

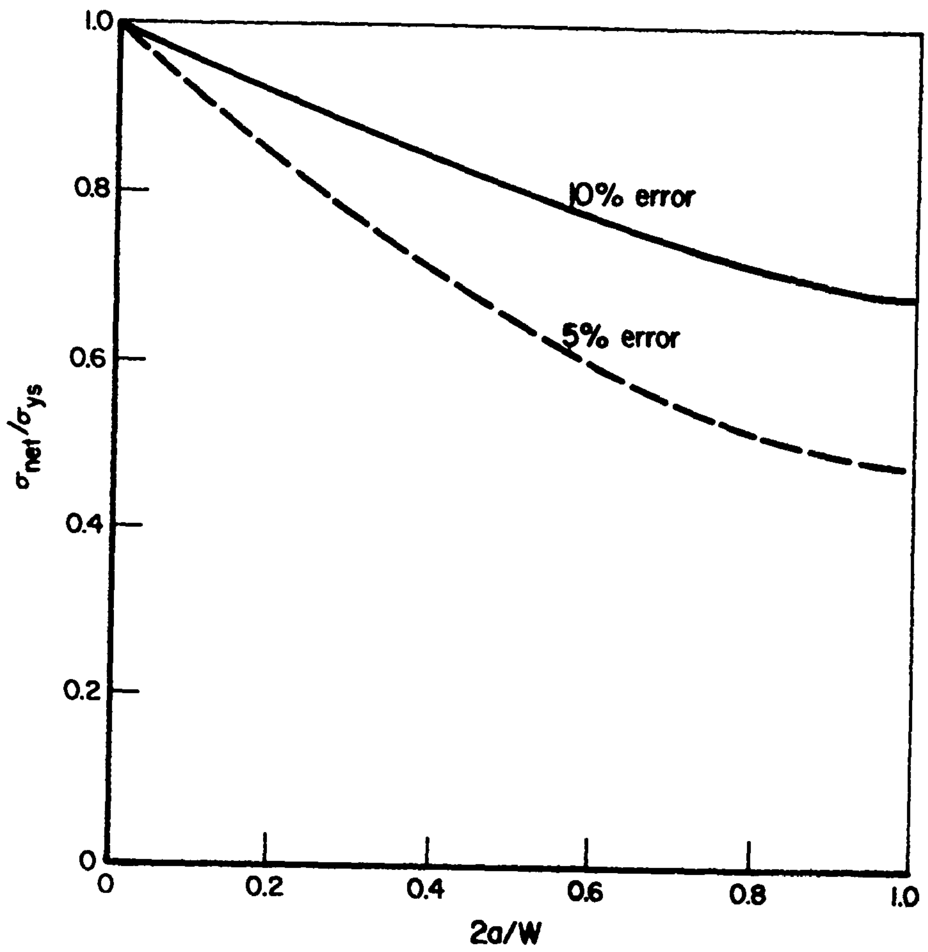


FIGURE 5.14. DIFFERENCE BETWEEN EXACT SOLUTION AND EQUATION (5.14) FOR CENTER CRACKED PANEL (Fuhring [71])

rational incorporation of closure and residual stresses in a crack growth model that can be dealt with in a relatively easy way without grossly simplifying assumptions. It is worth considering whether this model can predict a small crack effect and, subsequently, whether it can be expanded to include the effect of previous larger scale plasticity.

5.4 Prospects for Analyses of the Short Crack Effect in Fatigue

The short crack effect in fatigue results from the inability of linear elastic fracture mechanics procedures to predict the growth rate of physically small cracks. Several different possible reasons exist for this effect. It is entirely conceivable, for example, that very small cracks are influenced by microstructural features of the material that are not addressed by the conventional continuum-based LEFM techniques. In addition, three-dimensional effects (as arise in conjunction with corner cracks) exist to compound the problem. In view of these difficulties, it is not likely that predictive techniques can soon be devised to address very small cracks. Nonetheless, it is likely that an intermediate regime exists where progress can be made. The crack sizes for this regime lie below the limit of validity for LEFM and above the point where heterogeneity on the micro-mechanical scale strongly influences the crack growth process. In this regime an elastic-plastic fracture mechanics approach to fatigue can be effective. By pursuing such an approach, most investigators are not expecting to completely erase the short crack effect. Rather, they expect simply to shift the short crack demarcation point downwards by broadening the applicability to the limit of a continuum mechanics approach.

Considerable experience has been obtained in the past few years with elastic-plastic fracture mechanics--see, for example, reference [162]. This work has revealed the distinctive role played by the crack tip opening displacement (CTOD) during crack growth. While comparable results have not been obtained for subcritical cracking processes, it is tempting indeed to suppose that the CTOD may also serve as a rate controlling parameter there. This supposition is buttressed by the fact that, because of the relation that exists between CTOD and K under LEFM conditions, all K-based fatigue crack growth relations can be equally well expressed in terms of CTOD. If LEFM conditions are satisfied, there would be no difference between the two approaches. But, when LEFM conditions are not met, if the two approaches differ, on the basis of the elastic-plastic work, it is likely that the CTOD-based is the more correct.

To implement a CTOD-based fatigue crack growth approach it is necessary to quantitatively determine CTOD values for various conditions. A Dugdale crack model offers appropriate results in a convenient way. Results obtained using this model strongly indicate that the basic approach that has been suggested is correct [164]. Nevertheless, it should be clear that the use of a Dugdale model is merely an expedient. A general approach cannot rely upon the use of the Dugdale model, per se.

The Dugdale model, as used in [164], is entirely within the domain of liner elasticity. While nonlinear effects can be incorporated (e.g., by modeling the residual plasticity in the wake of this crack), this has not yet been done in an entirely satisfactory way. The success that has so far been achieved only indicates that the effort is aimed in the right direction. Further work to take into account crack closure and other nonlinear effects can be expected to further reduce the short crack effect; at least to the extent that continuum mechanics treatments can be expected to apply.

6.0 DISCUSSION

6.1 General

There are numerous reports in the literature that show two anomalous behaviors for crack growth rate data taken for short cracks. These are (1) considerable scatter about the long crack growth rate trend data, and (2) an apparent higher-than-expected growth rate (including growth at values of ΔK below the threshold). These have been termed the short crack effect. Throughout this review, reference has repeatedly been made to data that do not correlate when growth rate is plotted against the stress intensity factor, the common LEFM parameter. But, in many of these cases, the failure may have been due to the way LEFM was implemented rather than to some inherent deficiency in the theory. For example, K has been used for short cracks where closure is a factor, even though it is well accepted that ΔK alone does not account for closure of long cracks. This kind of example illustrates why it cannot be conclusively stated that failure to consolidate short crack data is due solely to shortcomings in LEFM. On the other hand, there are also examples in the literature of data for which the underlying assumptions of LEFM have been violated so that LEFM cannot apply.

There are factors other than the applicability of LEFM that are of consequence to the short crack effect. Data from the literature show that a host of both micro and macro mechanisms of the flow and cracking processes influence the crack growth rate. Differences in the cracking process, particularly near threshold, are especially numerous. They include multiple growth modes, combinations of modes, and the three-dimensional nature of the fracture process [42,79,146,147]; the length and configuration of the crack front involving dimensions of both the specimen and the microstructure [95,197]; anisotropy [57]; free surface effects on slip character including effects of surface treatment and crystallographic growth [146,147,184]; multiple cracking processes including possible environmental effects [124,144,170]; ragged crack fronts [189]; and finally, transient effects due to inclusions, grain boundaries, and grain-to-grain misorientation [108,131,180].

The factors just listed, which might be called materials related factors, have proponents who consider these to be most important factors. There is another school which believes that the most important factors are mechanics related. They credit the short crack effect to the influence of the plastic zone to crack length ratio on LEFM, anisotropic effects, surface residual stress and local closure effects due to plane stress surface flow confined by a plane strain surrounding, crack bifurcation and ill-defined crack fronts, stress redistribution due to notch root yielding and to material transient deformation behavior, and macroscopic closure due to residual stresses and deformations. It may be that a preference for the mechanics factors or the materials factors as being the most important in long and short crack behavior is only a function of training and background. But the truth is that all of these factors interrelate and must be considered together. Thus, for example, grain size should not be considered alone unrelated to yield strength, and conversely.

The process of going from a situation in which there is no crack on a scale on the order of the microstructure to a situation in which a crack exists, is transient. The crack tends to a steady state condition, the limit of which is the long crack condition. During this transition the crack length can be small compared to the size of the crack tip plastic zone that is required to sustain a growing crack [55]. In this regime, short cracks violate the confined plasticity requirements of LEFM. Thus, LEFM is invalid. While this kind of argument about LEFM is true at the microstructural scale, it is vacuous in an application to a single crystal. Yet a crack in a single crystal must still go through the transient phase. This is not to argue that LEFM is applicable to single crystals. To the contrary, it is to emphasize the transient nature of the process and note that it cannot be modeled by steady state theories such as LEFM.

The mechanism of initiation will control the length that the crack must attain before a steady state develops at its tip. Brittle intiations at inclusions localize the process and allow a sharp crack with a well defined tip and a continuous front to form. In contrast, ductile initiation involves a good deal of flow before slip band decohesion, for example, initiates a crack; often crystallographically and with a poorly defined tip and discontinuous front. Certainly, the second situation described will have to grow some to sharpen, etc., and thus will not achieve a steady state as quickly as its brittle counterpart.

Brittle initiation tends to form a crack which grows stably from the beginning, with limited flow at the crack tip. In contrast, ductile initiation would initially tend to violate the plastic zone to crack length limitation of LEFM. Results of the literature indicate that active plastic zones in the ductile case are as large as 0.3 mm while those for the brittle case approach 10^{-3} mm [41]. In this respect, a brittle steady state exists soon after inclusions crack, at crack lengths as small as can be consistently resolved using even highly sophisticated measurement systems. Ductile steady state by contrast develops only after extensive cracking. Significantly, LEFM criteria are satisfied for the lower extreme of brittle initiation at a crack length of about 10 μm--about the lower limit of detection. In contrast, LEFM criteria are violated at the upper limit of ductile initiation for cracks nearly 3 mm long.

For short cracks for which the plasticity requirements of LEFM are met (nominally brittle initiation), the literature suggests that metallurgical features are a controlling factor for cracks smaller than a few grains. Micromechanics is also a factor in this case in that local closure occurs due to plane stress flow on the surface that is contained within an unyielding plane strain field.† The growth of these cracks is strongly influenced by the transients associated with the mode and mechanism of initiation. Whether or not multiple initiation and/or branching occurs is also a factor. For this reason, it is expected that naturally initiated cracks will show a short crack effect; whereas, artificially induced cracks will demonstrate it to a much lesser degree, or not at all.

Smooth specimen data show the short crack effect inconsistently, and then only to a limited extent at finite growth rates. In view of the

† "Local closure" cannot be of consequence in ductile materials at stress levels that lead to bulk plasticity. The significance of local closure is inferred from the fact that data which show a "brittle" intergranular initiation do not show the short crack effect, whereas a "ductile" transgranular mechanism does [95].

preceding discussion, the limiting case of brittle initiation would not be expected to show any transient behavior and only limited closure because of limited flow. This is in fact observed [148]. As more inelastic action occurs, these effects should and do become more pronounced [179]. Thus, observed inconsistent nature of the short crack behavior is really not surprising, nor is it necessarily indicative of scatter. Near the threshold the short crack effect is more widespread and consistent, an observation that can be rationalized easily in terms of both microstructural effects and mechanics (local closure). However, it has been demonstrated that the threshold may be wiped out under variable amplitude (service like) loadings [78]. In this respect the fact that the effect develops consistently near the threshold may be of little practical concern.

To be unquestionably accurate, RFC analyses should make use of small crack thresholds. They may be substantially smaller than long crack thresholds. But, once the crack is growing, inclusion of any short crack behavior will have a negligible effect. This is particularly true in the case of brittle initiation. It is difficult to give a quantitative estimate of the short crack influence on life for smooth specimens. However, the case of confined flow at a notch has analogous ingredients. By integrating the growth rate behavior shown earlier in Figure 3.13, a decrease in life of about 10 percent is obtained for the most extreme of these results. The magnitude of this decrease depends on the difference in growth rate, the crack length over which the short crack rates act, and the duration of the ensuing long crack propagation. Of course, the shorter the critical crack size, the shorter the LEFM growth and the more significant the short crack effect on life.

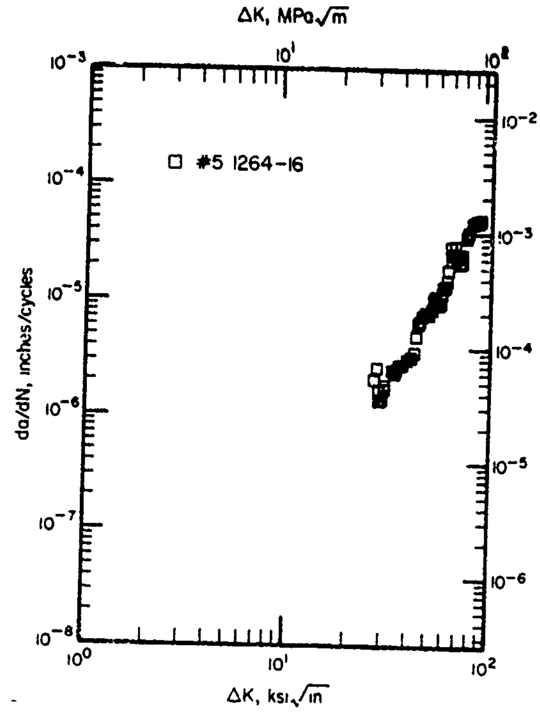
In the case of ductile initiation, both small and larger cracks may initially violate the LEFM confined plasticity requirement because of large crack tip plastic zones. That plastic zone may be due only to the initiation process. It may also be due to yielding at a notch. In the first case, the crack cannot behave as a long crack until it has grown beyond the initiation zone and has developed its own steady state field. Again both micromechanics and metallurgical features are important considerations in regard to the transient growth process. Equally important are multiple initiation and branching. Again, because artificial flaws would tend to concentrate deformation and tend to cause a more brittle initiation, natural cracks are expected to show the short crack effect much more so than artificial preflawed samples. When the plastic zone is due to notch inelastic action, not only may the contained plasticity requirement be violated, but the K solution is inappropriate. Cracks growing under this condition tend to be controlled primarily by the mechanics of the displacement controlled inelastic zone.

Unlike the case of confined crack tip plasticity, integration of short crack growth rates for cracks growing at inelastically strained notch roots suggests large errors could develop. Data in Reference [190] give nonconservative errors of more than an order of magnitude when LEFM data and analyses are used in place of the observed trends. Clearly, RFC analyses need to use the appropriate short crack threshold and growth rates in this case.

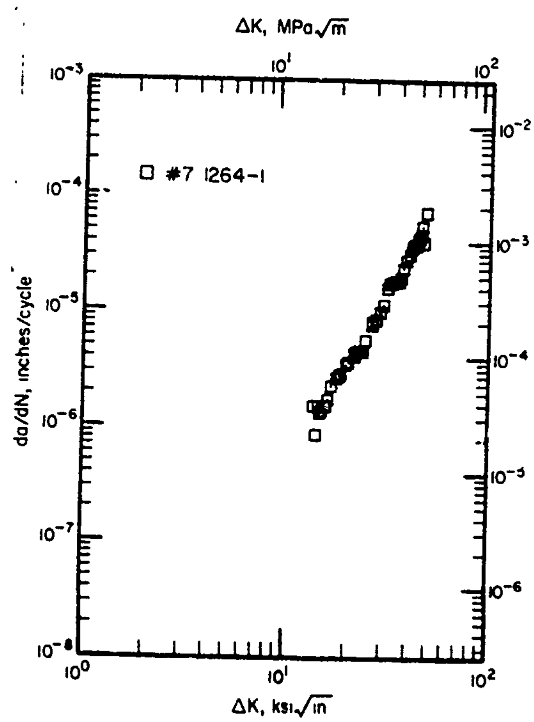
6.2 Engine-Specific Considerations

Because engine materials tend to be fine-grained high-strength materials, a trend toward brittle initiation can be anticipated in low stressed regions free of creep. Under these conditions, the influence of the short crack effect integrated over the growth of the crack may not be very significant, particularly if the critical crack size is relatively large. However, in high stressed areas subjected to a high temperature environment, a trend to more ductile initiation, particularly in the presence of creep, can be expected. In contrast to the brittle situation, the integrated influence of the short crack effect over the growth of the crack may be significant, even if the critical crack size is relatively large. Data developed by the General Electric Company (GE) [199], do show very limited anomalous growth trends in certain of their da/dN-ΔK results for IN718. Examples of this are shown for artificially flawed samples of IN718 in Figure 6.1, which presents only a portion of the GE data.

The trend of crack length with stress has been estimated from thresholds extracted from the data in Figure 6.1 and corresponding estimates of endurance limits provided by GE [199]. As noted in discussions of Figures 3.1 and 3.2, the LEFM trend has a slope of -1/2 on log - log coordinates. The value of K_{th} dictates the intercept. The upper bound to this trend, as

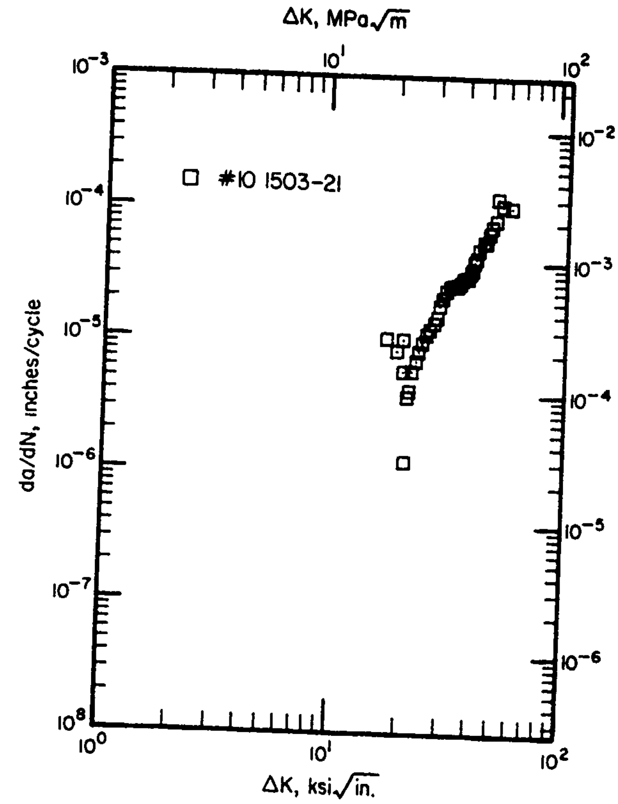


a. 800 F, R = -0.33, f = 0.33 Hz, LPT forging



b. 1000 F, R = 0, f = 0.33 Hz, LPT forging

FIGURE 6.1. FATIGUE CRACK PROPAGATION BEHAVIOR IN SURFACE PREFLAWED SAMPLES OF IN718 (GE DATA [201])



c. 1200 F, R = 0, f - 0.33 Hz, LPT forging

FIGURE 6.1. (Continued)

suggested by Staehle [75] and Smith [61], is the endurance limit stress. The interaction of these behaviors defines an estimate of the size below which LEFM will overestimate growth rate behavior. The resultant trends developed for IN718 based on the GE data are shown in Figure 6.2. Observe that for coarse grained (GS $\leq$ ASTM 4) materials, a short crack behavior could be expected for cracks as long as 1000 μm. In contrast, the results suggest that fine grained (GS = ASTM 10) material would show the effect for cracks up to 250 μm long. Given the fact that the data shown in Figure 6.1 involve initial artificial flaws typically 500 to 700 μm long, the absence of a significant short crack effect in these data is not surprising.

Another study has been made to examine short crack growth in an IN100 engine material [95]. That study indicated a major effect of microstructure and the character of the crack front. But, except for the influence of crack front, the results showed LEFM provided a viable basis for data correlation for cracks initially about 100 μm to 200 μm long. This length is approximately equal to the limit beyond which no short crack effect should arise, in view of Equation 4.5 and typical data for IN100.

The subject of short cracks to this point has been discussed without reference to environmental effects. However, engine hot path components exist in an aggressive gaseous environment and some comment on environmental interactions with short cracks is appropriate. As with ambient data, results for engine materials and environments are limited. The discussion, therefore, will focus on data for other material environment systems.

Consideration of the nature of environmentally assisted cracking (EAC) suggests that environmental short crack effects may arise from several sources. First, many environmental processes generate reaction products which can serve to block small, tight surface cracks. This will prevent entry of the aggressive species to the crack tip and retard the process. Alternatively, if these reaction products are mechanically stiff and are resistant to compression fracture, they may serve to wedge open crack [201]. This will locally reduce the range of the applied stress intensity factor, and thereby reduce at least the mechanical component of the growth rate. A second issue of consequence is the ready accessability of short crack tips to the environment. In processes where transport of the aggressive species is of

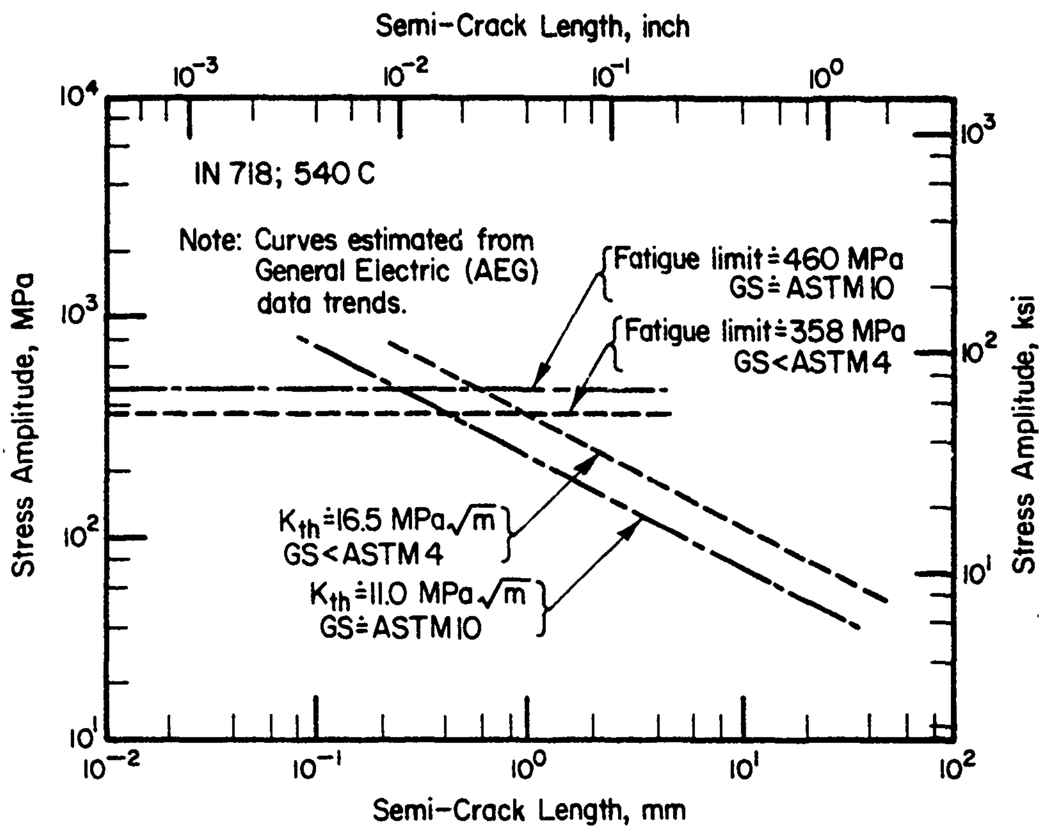


FIGURE 6.2. STRESS RANGE AS A FUNCTION OF SURFACE CRACK LENGTH FOR IN718 (BASED ON UNPUBLISHED GE DATA [201])

consequence, this would tend to enhance the growth rate for shorter cracks (e.g., [67]. A number of parameters are significant in such situations, including CTOD and the nature of the environment. The final issue of consequence involves cracks whose length lies within the steady state environmental process zone that develops for long cracks. Clearly any crack smaller than the dimension of this zone will not be environmentally similar to its longer steady state counterpart.

Depending on environmental and mechanical factors, one would anticipate that environmentally affected small cracks may grow either slower or faster than their longer counterparts. Data in the literature show that wedging action develops for a variety of reaction products [183,201]. Of particular significance in engine applications are the hard oxides that develop on a range of engine materials. Such oxides possess mechanical properties which could effectively block and wedge cracks. Wedging would have its greatest effect when its thickness is on the order of the CTOD. Small cracks and threshold conditions thus are first to be susceptible to its effects. In view of results for wedging effects in long cracks in aqueous media [183], wedging may significantly reduce the growth rate under these susceptible conditions. Data in the literature also show that there is an environmental short crack effect that significantly increases growth rates [67,109]. The first evidence of this effect, presented in Figure 6.3, appears in print before much of the current concern for short cracks [67]. More recent studies verify such trends at threshold [109] and finite growth rates [160]. Unfortunately, studies done to date do not permit determination of whether transport or nonsteady state environmental processes account for this increased rate.

In summary, data available for engine materials tend not to show a significant short crack effect. However, these results have been developed using artificial preflaws whose length is sufficient to preclude observing the effect were it to occur. Furthermore, the possible influence of notch root inelastic action is absent, along with possible environmental effects. In this respect, there is a need to develop engine materials data for both smooth and notched specimens, under conditions designed to characterize the effect of short cracks, if it exists. But, there is also a need for simple experiments

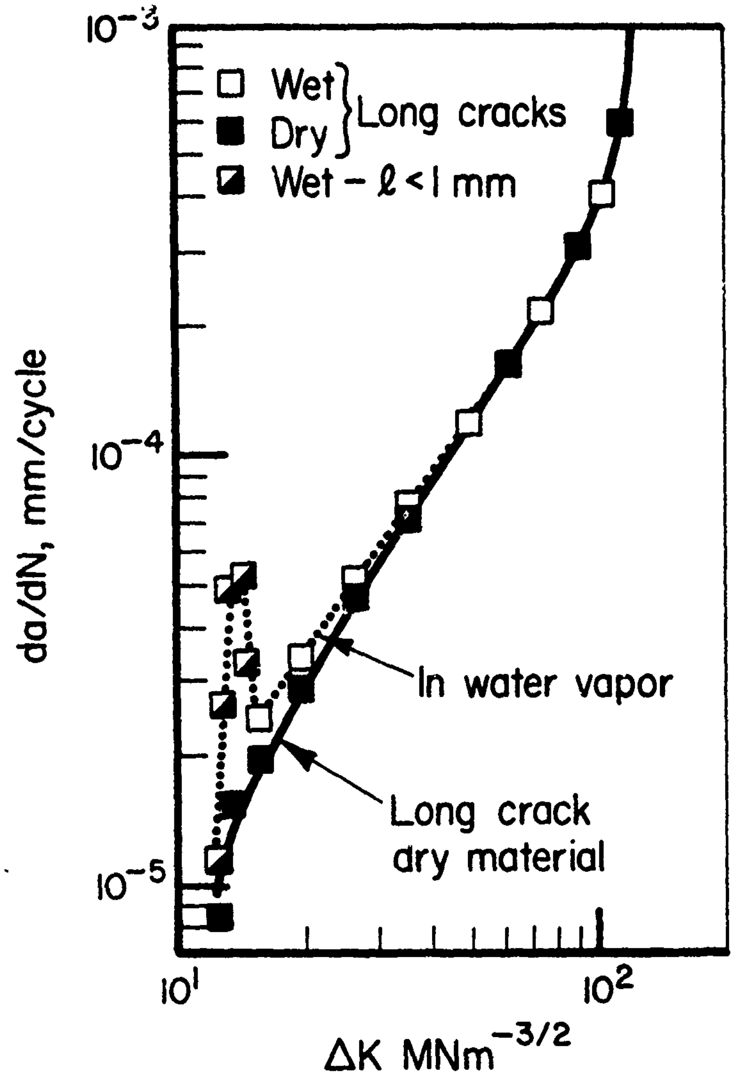


FIGURE 6.3. INFLUENCE OF ENVIRONMENT ON THE BEHAVIOR OF SHORT CRACKS (Holder [67])

that discriminate between the true limitations of LEFM and the failure to fully and correctly implement LEFM. Only after such data are developed can one assess the need for correlative models or additional mechanics analyses, and unravel the nature of possible environmental effects.

6.3 Critical Discriminating Experiments

This literature review has made it clear that there is a need for experiments which do two things:

- isolate conditions under which physically small cracks exhibit anomalous growth when properly analyzed via LEFM, and
- define those factors which control such growth in both smooth and notched specimens.

Since the effect appears to depend on the transient nature of the natural initiation process, naturally initiating cracks should be used. Furthermore, the effect appears to be fundamentally different at inelastically strained notch roots as compared to smooth specimens. Thus, both smooth and notched specimens should be used. Finally, differences in microcrack closure and fractographic features tend to most easily explain differences in short and long crack behavior in smooth specimens. Notched specimen behavior is likewise most easily explained by these factors, along with the fact that the local control condition is displacement control.

Discriminating tests that probe the influence of microcrack closure and fractographic transients would focus on smooth edge-cracked plate specimens precracked at their edges, to initiate well defined plane fronted cracks, i.e., a steady state crack. Use of a simple edge-cracked geometry coupled with selective testing at a range of R values would facilitate direct LEFM analysis of the data, thereby removing uncertainty in K calculation from the study. Depth of cracking should be controlled to develop long crack (steady state) fractography. To ensure repeatability, the depth and minimum section size should be controlled via some closed loop measurement system such as a crack gage. Various crack depths can then be achieved using the classic Frost approach of machining off the sample edges.

Tests should be restarted on one set of samples at the same set of loads, with enough samples tested over a range of loads to develop a range of plastic zone sizes at crack tips. To circumvent the influence of prior plastic zones, all samples should be stress relieved with care taken to avoid grain refinement or growth. Growth rate should be monitored and fractographic features studied after the test to ensure a steady state mode. These tests provide a closure-free, transient-free reference for all subsequent smooth specimens. They define the influence of plastic zone size to crack length ratio, in that a range of stress and crack lengths will be employed.

The influence of transients in growth due to differing growth mechanisms can be identified by running a corresponding set of experiments with naturally initiating and comparing their growth against the steady state data. The effect of m microcrack closure can be identified by selective testing to a similar matrix and thereafter removing material in the wake of the crack. Finally, the influence of grain size on fractrographic features and microcrack closure can be studied by selectively performing certain of the above experiments on materials heat treated to refine or enlarge the grains, and thereafter cold worked to yield the same strength level for all microstructures.

A similar series of selective discriminating tests should be run on notched samples. Here the center notched panel is appropriate. Different thicknesses can be studied to facilitate an examination of the influence of crack front length and geometry. Microcrack closure can be examined again by cutting away material in the wake of the crack.* Selective testing at a range of stresses and crack lengths facilitate direct study of the influence of notch plasticity. Since the influence zone of notch plasticity should scale with the linear scaling of the specimen planform, several geometrically similar samples of different relative size should be considered. Again, with the exception of the tests designed to study the corner crack configuration, the geometry chosen permits simple LEFM calculations.

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* The advantage here is that K before and after cutting is the same so that growth rate trends provide a clear-cut assessment of this factor. (Samples do not have to be heat treated to remove the prior plastic zone.)

Regarding definition of potential problems in engine materials, several edge-cracked plate specimens of engine materials should be tested to define the range of crack lengths susceptible and to determine the extent of the effect at typical service temperatures. Much of the discriminating program discussed above could be done using engine materials. However, the fact that they cannot be reheat treated without microstructural changes suggests that the results would be clouded by an uncontrolled variable.

7. SUMMARY

This review suggests that situations defined as short crack problems can be ascribed to a wide number of factors. Included are stress ratio effects (microcrack closure); violation of the limitations of LEFM; inappropriate, incorrect, or incomplete implementation of LEFM; and transients due to initiation (modes and mechanisms of growth, sharpness and definition of crack front, etc.). Smooth and notched specimens appear to be affected in various ways by all of these, depending on the specimen and crack geometry and the material. Notched specimens have the additional unique complication that the local field controlling initiation and microcrack growth at the notch root may be different than that imposed by the far field. Environment appears to be a factor, but its influence is often clouded by a number of other factors. Significantly, there is no single or explicit length bound above which a short crack effect will not develop.

It has been shown that the phenomenological data are unclear as to which factors are of consequence and why. Correlative and analytical models exist to predict the behavior of short cracks. However, because the causes of the anomalous growth are uncertain, it is not clear which of these is most useful in a RFC analysis. Analytical models exist specifically for smooth specimens which contain small cracks whose plastic zone is large compared to its length. All are based on pseudoplastic solutions that address the crack tip plasticity in terms of strip yield models. The CTOD has been suggested as a basis for correlating microcrack growth and it appears to be consistent with the phenomenological results. Analytical models, again based on strip yield models, have been proposed to deal with closure. These appear to be consistent with long crack trends and are not incompatible with results for physically small cracks. It appears that cracks at notches remain as the major area of uncertainty in terms of simple analytical models. Of course, since all analytical models are simple first-generation formulations, there is still much room for improvement even for the smooth specimen geometries.

It is concluded that the short crack effect arises primarily because of crack tip plasticity, transients from the initiation process, and incorrect or incomplete implementation of LEFM. The phenomenological data tend not to discriminate between these. Consequently, it is not now certain which of these is significant, when, and why. Thus, discriminating tests are required. The CTOD appears to be a viable basis to track the growth of small (short) cracks. Its use as a measure of the crack driving force warrants continued study.

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